Errata, Comments, and Additional Solutions

Student Solution Manual

Includes all errata and comments through March 8, 2005

We thank Lewis Atchison, Jonathan Bergknoff, Chris Hruska, Todd Kemp, Steven Hoffenson, Dick Palas, James Versyp, and Peng Zhao for pointing out errors.

Page 1 Remark concerning Exercise 0.2.1: The reference to Fermat's little theorem should be deleted.

For the solution to Exercise 0.4.7 to be correct, one would have to change the problem to exchange h and k (i.e., $h : C \to A, k : A \to C$). The solution for the existing exercise is

The following are well defined: $g \circ f : A \to C$, $h \circ k : C \to C$, $k \circ g : B \to A$, $k \circ h : A \to A$, and $f \circ k : C \to B$. The others are not, unless some of the sets are subsets of the others. For example, $f \circ g$ is not because the range of g is C, which is not the domain of f, unless $C \subset A$.

Exercise 0.4.9 part (a) gives the solution for $a = 2$, whereas the exercise stipulated $a = 3$. The correct solution is $f\big(g(h(3))\big) = f\big(g(-1)\big) = f(-3) = 8$.

Page 5 The solution in Exercise 0.7.9 is the solution to the wrong equation, $x^2 + 2ix - 1 = 0$. The correct solution is

(a) The quadratic formula gives $x = \dfrac{-i \pm \sqrt{-1-8}}{2}$, so the solutions are $x = i$ and $x = -2i$.

The solution to part (c) is missing. It is

(c) Multiplying the first equation through by $(1 + i)$ and the second by i gives

$$\begin{aligned} i(1+i)x - (2+i)(1+i)y &= 3(1+i) \\ i(1+i)x - \qquad\qquad\quad y &= 4i, \end{aligned}$$

which gives

$$-(2+i)(1+i)y + y = 3 - i, \quad \text{i.e.,} \quad y = i + \frac{1}{3}.$$

Substituting this value for y then gives $x = \dfrac{7}{3} - \dfrac{8}{3}i$.

Page 9 The (c) in the first line should be at the beginning of the third line.

Page 11 The solution to Exercise 1.3.15 uses the dot product, which is introduced only in the next section. Here is a different solution:

We need to show that $A(\vec{\mathbf{v}} + \vec{\mathbf{w}}) = A\vec{\mathbf{v}} + A\vec{\mathbf{w}}$ and that $A(c\vec{\mathbf{v}}) = cA\vec{\mathbf{v}}$. By Definition 1.2.4,

$$(A\vec{\mathbf{v}})_i = \sum_{k=1}^{n} a_{i,k}v_k, \quad (A\vec{\mathbf{w}})_i = \sum_{k=1}^{n} a_{i,k}w_k, \text{ and}$$

$$\big(A(\vec{\mathbf{v}} + \vec{\mathbf{w}})\big)_i = \sum_{k=1}^{n} a_{i,k}(v+w)_k = \sum_{k=1}^{n} a_{i,k}(v_k + w_k) = \sum_{k=1}^{n} a_{i,k}v_k + \sum_{k=1}^{n} a_{i,k}w_k = (A\vec{\mathbf{v}})_i + (A\vec{\mathbf{w}})_i.$$

Similarly, $\big(A(c\vec{\mathbf{v}})\big)_i = \sum_{k=1}^{n} a_{i,k}(cv)_k = \sum_{k=1}^{n} a_{i,k}cv_k = c\sum_{k=1}^{n} a_{i,k}v_k = c(A\vec{\mathbf{v}})_i$.

Page 12 Exercise 1.4.1: If the matrix A consists of a single row, then $A\vec{\mathbf{v}}$ is a number.

Page 13 Exercise 1.4.23, part (a): The first term in the cited formula is 1^2, not a^2:

$$1^2 + 2^2 + \cdots n^2 = \frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}$$

not

$$a^2 + 2^2 + \cdots n^2 = \frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}.$$

The solution to Exercise 1.5.3 uses the word infimum, not yet introduced. It means the same thing as greatest lower bound, Definition 0.5.2.

Page 16 Part (d) of Solution 1.5.17 uses the property that the sequence $\mathbf{a}_m$ converges, but the hypothesis of Theorem 1.5.16 (d) states that the sequence is bounded; it doesn't say that it converges.

The solution should be:

(d) Find C such that $|\mathbf{a}_m| \leq C$ for all m; saying that $\mathbf{a}_m$ is bounded means exactly that such a C exists. Choose $\epsilon > 0$, and find M such that when $m > M$, then $|c_m| < \epsilon/C$ (this is possible since the c_m converge to 0). Then when $m > M$ we have

$$|c_m\mathbf{a}_m| = |c_m||\mathbf{a}_m| \leq \frac{\epsilon}{C}C = \epsilon.$$

Page 20 Solution 1.7.3, part (a): the last term should be $2x - \sin x$, not $2x + \cos x$:

$$f'(x) = \Big(3\sin^2(x^2 + \cos x)\Big)\Big(\cos(x^2 + \cos x)\Big)\Big(2x - \sin x\Big).$$

Page 24 Solution 1.9.1: the second displayed equation should have $\sin 1/h$, not $\sin h$:

$$\lim_{h\to 0} \frac{g(h) - 0}{h} = \lim_{h\to 0} h \sin\frac{1}{h} = 0.$$

page 26 Solution 1.9, part b: the matrix multiplication is wrong! This should be: The matrices of the compositions are given by matrix multiplication:

$$[S \circ T] = [S][T] = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad \text{and} \quad [T \circ S] = [T][S] = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}.$$

Page 27 Solution 1.17: the first entry of the vector $A\vec{\mathbf{v}}_1$ is $\frac{\sqrt{2}}{2}$, not $\frac{2}{\sqrt{2}}$.

Page 28 Solution 1.27: The solution for part (b) is really a second way of solving part (c). Here is the correct solution for part (b):

(b) This function is not differentiable. If you set $g(t) = \begin{pmatrix} t \\ t \end{pmatrix}$, then $f \circ g(t) = 2|t|$ is not differentiable at $t = 0$ but g is differentiable at $t = 0$, so f is not differentiable at the origin, which is $g(0)$. (If f were differentiable, then by the chain rule the composition would be differentiable, and it isn't.)

Page 32 Solution 2.1.9: There is a second invisible 1: second row, third entry.

Page 38 Part (b) of Solution 2.4.13 starts at the top of the page. The approximations to $\int_0^1 \frac{dx}{1+x} = \log 2 = 0.69314718055995\ldots$ obtained with the coefficients

$$\begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix}, \quad \begin{bmatrix} 1/6 \\ 2/3 \\ 1/6 \end{bmatrix}, \quad \begin{bmatrix} 1/8 \\ 3/8 \\ 3/8 \\ 1/8 \end{bmatrix}$$

are .75 for $n = 1$, $\frac{25}{36} = .6944\ldots$ for $n = 2$, and $\frac{111}{160} = .69375$ for $n = 3$.

Page 44 First line of Solution 2.6.5: The ith column of $[R_A]$ is $[R_A]\vec{\mathbf{e}}_i$, not $[R_A]\vec{\mathbf{e}}_1$.

Page 53 Solution 2.9.17, part (c): "If there were," not "If these were."

page 55 Solution 2.1 is wrong. It should be:

(a) Row reduction gives

$$\begin{bmatrix} 1 & 1 & -1 & a \\ 1 & 0 & 2 & b \\ 1 & a & 1 & b \end{bmatrix} \quad \rightarrow \quad \begin{bmatrix} 1 & 1 & -1 & a \\ 0 & -1 & 3 & b-a \\ 0 & a-1 & 2 & b-a \end{bmatrix}$$
$$\rightarrow \quad \begin{bmatrix} 1 & 0 & 2 & b \\ 0 & 1 & -3 & a-b \\ 0 & 0 & 3a-1 & a(b-a) \end{bmatrix}.$$

We will consider separately the cases $a = 1/3$ and $a \neq 1/3$. If $a = 1/3$, we get

$$\begin{bmatrix} 1 & 0 & 2 & b \\ 0 & 1 & -3 & \frac{1}{3}-b \\ 0 & 0 & 0 & \frac{1}{3}(b-\frac{1}{3}) \end{bmatrix},$$

so if $a = 1/3$ and $b = 1/3$, there are infinitely many solutions (there is no pivotal 1 in the 4th column or the 3rd column). If $a = 1/3$ and $b \neq 1/3$, then there are no solutions: by futher row reducing we can get a pivotal 1 in the 4th column.

If $a \neq 1/3$, then we can further row reduce our original matrix to get a pivotal 1 in the 3rd column. In that case the system of equations has a unique solution.

(b) We have already done all the work: the matrix of coefficients (i.e., the matrix consisting of the first three columns) is invertible if and only if $a \neq 1/3$.

Page 56 Solution 2.11, part (a): The subspace has dimension 2, not 3.

Page 59 Solution 2.27: In the third displayed formula, $|\vec{\mathbf{w}}||\top$ should be $|\vec{\mathbf{w}}^\top|$. The second displayed equation also has problems, because the dimensions aren't right. We aren't assuming that A is square; if it is $m \times n$, then for $A\vec{\mathbf{v}}$ to make sense $\vec{\mathbf{v}}$ must be a vector in $\mathbb{R}^n$. But then $A^\top$ is $n \times m$ so $A^\top \vec{\mathbf{v}}$ does not make sense.

Here is a corrected solution:

First, note that

$$|A| = \sup_{|\vec{\mathbf{v}}|=1} |A\vec{\mathbf{v}}| = \sup_{|\vec{\mathbf{w}}|=1, |\vec{\mathbf{v}}|=1} |\vec{\mathbf{w}}^\top||A\vec{\mathbf{v}}| \geq \sup_{|\vec{\mathbf{w}}|=1, |\vec{\mathbf{v}}|=1} |\vec{\mathbf{w}}^\top A\vec{\mathbf{v}}|; \tag{1}$$

similarly,

$$|A^\top| = \sup_{|\vec{\mathbf{w}}|=1} |A^\top\vec{\mathbf{w}}| = \sup_{|\vec{\mathbf{v}}|=1, |\vec{\mathbf{w}}|=1} |\vec{\mathbf{v}}^\top||A^\top\vec{\mathbf{w}}| \geq \sup_{|\vec{\mathbf{w}}|=1, |\vec{\mathbf{v}}|=1} |\vec{\mathbf{v}}^\top A^\top\vec{\mathbf{w}}|. \tag{2}$$

Note also that

$$|\vec{\mathbf{w}}^\top A\vec{\mathbf{v}}| = |\big((A\vec{\mathbf{v}})^\top\vec{\mathbf{w}}\big)^\top| = |(A\vec{\mathbf{v}})^\top\vec{\mathbf{w}}| = |\vec{\mathbf{v}}^\top A^\top\vec{\mathbf{w}}|.$$

(Since $(A\vec{\mathbf{v}})^\top\vec{\mathbf{w}}$ is a number, it equals its transpose.)

Therefore, if we can show that the inequalities (1) and (2) are equalities for appropriate $\vec{\mathbf{v}}, \vec{\mathbf{w}}$, we will have shown that $|A| = |A^\top|$.

By definition,

$$\|A\| = \sup_{|\vec{\mathbf{v}}|=1} |A\vec{\mathbf{v}}|,$$

and since the set $\{\, \vec{\mathbf{v}} \in \mathbb{R}^n \mid |\vec{\mathbf{v}}| = 1 \,\}$ is compact, the maximum is realized: there exists a unit vector $\vec{\mathbf{v}}_0$ such that $|A\vec{\mathbf{v}}_0| = \|A\|$. Let $\vec{\mathbf{w}}_0 = \dfrac{A\vec{\mathbf{v}}_0}{|A\vec{\mathbf{v}}_0|}$. Then

$$\|A\| = |A\vec{\mathbf{v}}_0| = \underbrace{\vec{\mathbf{w}}_0^\top\vec{\mathbf{w}}_0}_{1}|A\vec{\mathbf{v}}_0| = \vec{\mathbf{w}}_0^\top \frac{A\vec{\mathbf{v}}_0}{|A\vec{\mathbf{v}}_0|}|A\vec{\mathbf{v}}_0| = \vec{\mathbf{w}}_0^\top A\vec{\mathbf{v}}_0 = |\vec{\mathbf{w}}_0^\top A\vec{\mathbf{v}}_0|.$$

Therefore,

$$\|A\| = \sup_{|\vec{\mathbf{w}}|=1, |\vec{\mathbf{v}}|=1} |\vec{\mathbf{w}}^\top A\vec{\mathbf{v}}|$$

and by a similar argument,

$$|A^\top| = \sup_{|\vec{\mathbf{w}}|=1, |\vec{\mathbf{v}}|=1} |\vec{\mathbf{v}}^\top A^\top\vec{\mathbf{w}}|.$$

It follows that $|A| = |A^\top|$.

Page 61 Solution 3.1.1 shows that the unit circle is a smooth curve; the exercise asked you to show that the unit sphere is a smooth surface:

The derivative of $F\begin{pmatrix} x \\ y \end{pmatrix} = x^2 + y^2 + z^2 - 1$ is $[2x \quad 2y \quad 2z]$, which is $[0 \quad 0 \quad 0]$ only at the origin, which does not satisfy $F\begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0$. So the unit sphere given by $x^2 + y^2 + z^2 = 1$ is a smooth surface.

Page 62 Solution 3.1.9 includes some language introduced in Section 3.6:

The critical point $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ is a saddle point, where p goes up and q goes down. The other critical points are $x = 0,\ y = \pm 1/\sqrt{2}$, which are minima.

The solution is complete without it.

Page 72 Solution 3.3.13 has an error on the last line; the $\frac{1}{4}uv$ should be deleted.

More serious, in the given solution we are using techniques from Section 3.4 Here is a new solution, using the techniques of Section 3.3.

3.3.13 First, we compute the partial derivatives and second partials:

$$D_{(1,0)}f = \frac{1+y}{2\sqrt{x+y+xy}} \qquad D_{(0,1)}f = \frac{1+x}{2\sqrt{x+y+xy}}$$

$$D_{(2,0)}f = \frac{\sqrt{x+y+xy}\cdot 0 - (1+y)\frac{1+y}{2\sqrt{x+y+xy}}}{2(x+y+xy)} = \frac{-(1+y)^2}{4(x+y+xy)^{3/2}}$$

$$D_{(1,1)}f = \frac{\sqrt{x+y+xy}\cdot 1 - \frac{(1+y)(1+x)}{2\sqrt{x+y+xy}}}{2(x+y+xy)} = \frac{2(x+y+xy)-(1+y)(1+x)}{4(x+y+xy)^{3/2}}$$

$$D_{(0,2)}f = \frac{\sqrt{x+y+xy}\cdot 0 - (1+x)\frac{1+x}{2\sqrt{x+y+xy}}}{2(x+y+xy)} = -\frac{(1+x)^2}{4(x+y+xy)^{3/2}}.$$

At the point $\begin{pmatrix} -2 \\ -3 \end{pmatrix}$ these are

$$D_{(1,0)}f\begin{pmatrix} -2 \\ -3 \end{pmatrix} = \frac{1-3}{2\sqrt{-2-3+6}} = -1,$$

$$D_{(0,1)}f\begin{pmatrix} -2 \\ -3 \end{pmatrix} = \frac{1-2}{2\cdot 1} = -\frac{1}{2},$$

$$D_{(2,0)}f\begin{pmatrix} -2 \\ -3 \end{pmatrix} = \frac{-(1-3)^2}{4\cdot 1} = -1,$$

$$D_{(1,1)}f\begin{pmatrix} -2 \\ -3 \end{pmatrix} = \frac{2\cdot 1-(-2)(-1)}{4} = 0,$$

$$D_{(0,2)}f\begin{pmatrix} -2 \\ -3 \end{pmatrix} = -\frac{(-1)^2}{4\cdot 1} = -\frac{1}{4}.$$

Write $x = -2 + u,\ y = -3 + v$; i.e., the increment $\vec{\mathbf{h}}$ is $\begin{pmatrix} u \\ v \end{pmatrix}$. The Taylor polynomial of degree 2 of f at $\begin{pmatrix} -2 \\ -3 \end{pmatrix}$ is then $1 - u - \frac{1}{2}v + \frac{1}{2}(-u^2 - \frac{1}{4}v^2)$:

$$P^2_{f,\begin{pmatrix} -2 \\ -3 \end{pmatrix}}\begin{pmatrix} -2+u \\ -3+v \end{pmatrix} = \underbrace{1}_{f\begin{pmatrix} -2 \\ -3 \end{pmatrix}} + \underbrace{-u}_{D_{1,0}f\begin{pmatrix} -2 \\ -3 \end{pmatrix}u^1v^0} + \underbrace{-\frac{1}{2}v}_{D_{(0,1)}f\begin{pmatrix} -2 \\ -3 \end{pmatrix}u^0v^1} + \underbrace{-\frac{1}{2}u^2}_{\frac{1}{2}D_{(2,0)}f\begin{pmatrix} -2 \\ -3 \end{pmatrix}u^2v^0} + \underbrace{\frac{1}{2}\left(-\frac{1}{4}v^2\right)}_{\frac{1}{2}D_{(0,2)}f\begin{pmatrix} -2 \\ -3 \end{pmatrix}u^0v^2}.$$

Page 72 Two lines before Solution 3.4.1: "This shows that," not "this show that."

Page 73 Solution to Exercise 3.4.5: Several corrections are needed, in the part beginning "On the other hand":

In the displayed equation, the term beginning $4C$ should be $8C$. This leads to $a = 1/3$, $b + 4c = 8/3$, and $c = 1/3$. Thus the text should read

$$\int_0^{2h} f(t)\,dt = 2Ah + 4B\frac{h^2}{2} + 8C\frac{h^3}{3} + \int_0^{2h} R(t)\,dt.$$

Note that $\int_0^{2h} R(t)\,dt \in o(h^3)$. Thus we find

$$\begin{aligned} a + b + c &= 2 \\ b + 2c &= 2 \\ b + 4c &= \frac{8}{3} \end{aligned} \quad \text{with solution} \quad \begin{aligned} a &= \frac{1}{3} \\ b &= \frac{4}{3} \\ c &= \frac{1}{3} \end{aligned}.$$

Page 83 Solution 3.7.1: The first displayed equation should be

$$[\,4xy \quad 2x^2\,] = \lambda\,[\,4x + 3y \quad 3x\,], \quad \text{not} \quad [\,2xy \quad 2x^2\,] = \lambda\,[\,4x + 3y \quad 3x\,].$$

84 Solution 3.7.5: The computed volume should be multiplied by 8; what we computed is just the volume in the first octant, where $x \geq 0$, $y \geq 0$, $z \geq 0$. Thus the total volume is $4\sqrt{2}$.

Page 86 Solution 3.7.13, second paragraph: "stretches," not "streches," and (in the second line) "hyperbola," not "parabola."

Page 90 Solution 3.9.9, part 9c: Since the hypocycloid has four arcs, the length of one arc, which is what the exercise asked for, is $3a/2$.

Page 91 Solution 3.3, part (c): "computed in part (b)," not "computed in the solution to Exercise 3.3."

Page 92 Solution 3.11, part (a) : "implicit function theorem," not "implicit function." The $\begin{pmatrix} 0 \\ r \\ 0 \end{pmatrix}$ in the displayed equation should be $\begin{pmatrix} r \\ 0 \\ 0 \end{pmatrix}$:

$$D_z(y\cos z - x\sin z)\begin{pmatrix} r \\ 0 \\ 0 \end{pmatrix} = -y\sin z - x\cos z\begin{pmatrix} r \\ 0 \\ 0 \end{pmatrix} = -0\cdot\sin 0 - r\cos 0 = -r \neq 0.$$

(We are evaluating $D_z(y\cos z - x\sin z)$ at $\begin{pmatrix} r \\ 0 \\ 0 \end{pmatrix}$, not multiplying.)

Page 95 Solution 3.21: This is correct but the notation is inconsistent with the problem. We have rewritten the solution:

3.21 (a) Consider the figure below (left side). The cosine law says

$$e^2 = a^2 + d^2 - 2ad\cos\phi = b^2 + c^2 - 2bc\cos\psi.$$

FIGURE FOR SOLUTION 3.21. Left: Our quadrilateral. Right: An angle inscribed in a circle is half the corresponding angle at the center of the circle. This is key to proving that a quadrilateral can be inscribed in a circle if and only if opposite angles are supplementary, a result you may remember from high school.

(b) The area of the quadrilateral is given by $ad\sin\phi + bc\sin\psi$.

(c) Our quadrilateral satisfies the constraint

$$a^2 + d^2 - 2ad\cos\phi - b^2 - c^2 + 2bc\cos\psi = 0.$$

So the Lagrange multiplier theorem asserts that at the maximum of the area function, there is a number λ such that

$$[ad\cos\phi,\ bc\cos\psi] = \lambda[2ad\sin\phi,\ -2bc\sin\psi].$$

This immediately gives $\cot\phi + \cot\psi = 0$, i.e., opposite angles are supplementary (i.e., sum to $180°$). It follows from high school geometry that the quadrilateral can be inscribed in a circle. (The key statement used is that an angle inscribed in a circle is half the corresponding angle at the center of the circle, as shown to the right of the figure above.)

Page 109 Solution 4.5.17, first line: rth smallest, not largest.

Page 115 Five lines from the bottom and seven lines from the bottom: "property (d)" should be "property (4)."

Page 117 Exercise 4.10.3 should have been:

"Show that in complex notation, with $z = x + iy$, the equation of the lemniscate of Figure 4.10.3 can be written $|z^2 - \frac{1}{2}| = \frac{1}{2}$." The equation given in the text is the equation for a different lemniscate.

Solution for corrected exercise

Since $|z^2 - \frac{1}{2}| = -\frac{1}{2}$ has no solutions, squaring both sides of $|z^2 - \frac{1}{2}| = \frac{1}{2}$ does not add more points to the graph. Squaring both sides gives $|z^2 - \frac{1}{2}|^2 = \frac{1}{4}$, which can be rewritten

$$\frac{1}{4} = \left| r^2\cos^2\theta + ir^2\sin^2\theta - \frac{1}{2}\right|^2 = r^4\cos^2 2\theta - r^2\cos 2\theta + \frac{1}{4} + r^4\sin^2 2\theta,$$

which gives the desired result:

$$r^2(r^2 - \cos 2\theta) = 0, \quad \text{i.e.,} \quad r^2 = \cos 2\theta.$$

Correction to solution given in the solution manual, for the original exercise

In the first paragraph, "Since $-|z^2+1| = -1$ describes the same set of points as $|z^2+1| = 1$, squaring both sides does not add more points to the graph" should be

"Since $|z^2+1| = -1$ has no solutions, squaring both sides of $|z^2+1| = 1$ does not add more points to the graph."

Page 122 Exercise 4.11.1, first line: $|\mathbf{x}||^p$ should be $|\mathbf{x}|^p$.

Page 124 Solution 4.11.11: The solution in the manual is correct, but here is another solution, by Cornell student Vorrapan Chandee, then a freshman:

It is clearly enough to show the result for any monomial $x_1^{k_1} \cdots x_n^{k_n}$. By Fubini's Theorem, we have

$$\begin{aligned}\int_{\mathbb{R}^n} x_1^{k_1} \cdots x_n^{k_n} e^{-|\mathbf{x}|^2} \, |d^n\mathbf{x}| &= \int_{\mathbb{R}^n} x_1^{k_1} e^{-x_1^2} \cdots x_n^{k_n} e^{-x_n^2} \, |d^n\mathbf{x}| \\ &= \left(\int_{\mathbb{R}} x_1^{k_1} e^{-x_1^2} |dx_1| \right) \cdots \left(\int_{\mathbb{R}} x_n^{k_n} e^{-x_n^2} |dx_n| \right).\end{aligned}$$

The original integral exists if each integral in the above product exists.

So it suffices to show that the one-dimensional integral $\int_{\mathbb{R}} x^k e^{-x^2} |dx|$ exists for $k = 0, 1, 2, \ldots$. By a symmetry argument, we can reduce this to showing that $\int_0^{\mathbb{R}} x^k e^{-x^2} dx$ exists.

But $x^k e^{-x^2}$ is eventually dominated by e^{-x}, which is easily seen to be integrable over $[0, \infty)$.

Page 130 Solution 4.15, last line: πc^2 should be πu^2 (substituting $c = \pi/2$ in $2cu^2$).

Page 133 The solution to Exercise 5.1.3 is correct, but it does not use the hint given in the textbook. Here is a solution using the hint:

Set $T = [\vec{\mathbf{v}}_1, \ldots, \vec{\mathbf{v}}_k]$. Since the vectors $\vec{\mathbf{v}}_1, \ldots, \vec{\mathbf{v}}_k$ are linearly dependent, $\operatorname{rank} T < k$. Further, $\operatorname{Img} T^\top T \subset \operatorname{Img} T^\top$, so

$$\operatorname{rank} T^\top T \le \operatorname{rank} T^\top \underbrace{=}_{\text{Prop. 2.5.12}} \operatorname{rank} T < k.$$

Since $T^\top T$ is a $k \times k$ matrix with rank $< k$, it is not invertible, hence its determinant is 0, so

$$\operatorname{vol}_k P(\vec{\mathbf{v}}_1, \ldots, \vec{\mathbf{v}}_k) = \sqrt{\det T^\top T} = 0.$$

Page 135 Solution 5.2.5: In the second line of the remark, we use the notation $\gamma^* f$, which we haven't yet defined. It can be replaced by $f \circ \gamma$.

Page 136 In the solution to Exercise 5.3.5, part (a), we say that the integral can't be computed using Maple. We had been using version 5. With version 7, it can:

$$\int_0^{2\pi}\int_0^a 2r\sqrt{9+r^2(4\sin^2\theta+9\cos^2\theta)}\,dr\,d\theta = \frac{2}{3\sqrt{1+a^2}}\left(36\sqrt{\frac{1+a^2}{9+4a^2}}a^2\text{ellipticK}\sqrt{-5\frac{a^2}{9+4a^2}}\right.$$
$$+81\sqrt{\frac{1+a^2}{9+4a^2}}\text{elliptic}\pi\left(\frac{-5}{4},\sqrt{-5\frac{a^2}{9+4a^2}}\right)+16\sqrt{\frac{1+a^2}{9+4a^2}}a^4\text{ellipticE}\sqrt{-5\frac{a^2}{9+4a^2}}$$
$$\left.+36\sqrt{\frac{1+a^2}{9+4a^2}}a^2\text{ellipticE}\sqrt{-5\frac{a^2}{9+4a^2}}-9\pi\sqrt{1+a^2}\right).$$

(The elliptic functions "elliptic K," "elliptic E," and "elliptic π" are tabulated functions; tables with these functions can be found, for example, in *Handbook of Mathematical Functions*, edited by Milton Abramowitz and Irene Stegun (Dover Publications, Inc.).)

Page 144 Solution to Exercise 5.3.21 (which starts on page 143): the second line of the first displayed equation should be $\left(p^2s^{2p-1}+q^2s^{2q-1}+r^2s^{2r-1}\right)^2$, not $\left(p^2s^{2p-1}+q^2s^{2q-1}r^2s^{2r-1}\right)^2$.

Page 148 Next-to-last line: "bigger than the dimension," not "bigger the dimension."

Page 151 Solution 6.1.17, end of first paragraph of part (b): "spade", not "space".

Page 153 Solution 6.2.1, part (b): Second line of displayed equation, P^o should be P.

Page 154 In the first line, the three vectors should be written with square brackets and they should be enclosed in parentheses. Also, the numbers should be aligned.

Page 154 In Exercise 6.2.3, part (b), the last line has an extra "2". It should be

$$=\int_0^1\left(\int_{-(1-w)}^{1-w}\left(\int_{-\sqrt{(w-1)^2-v^2}}^{\sqrt{(w-1)^2-v^2}}2(u-v)(w-v)du\right)dv\right)dw.$$

Page 155 Solution 6.3.7: We wrote on page 231, "The tricky point when changing bases is not getting confused about what direction you are going. Going from new basis to old, use the change of basis matrix. Going from old to new, use its inverse." We got mixed up anyway. The change of basis matrix given for part (c) is actually the change of basis matrix for part (b), and vice versa.

Thus parts (b) and (c) should be

(b) The basis $\vec{\mathbf{w}}_1, \vec{\mathbf{w}}_2$ is also direct since

$$dx\wedge dz\left(\begin{bmatrix}2\\-3\\-4\end{bmatrix},\begin{bmatrix}1\\2\\5\end{bmatrix},\right)=\det\begin{bmatrix}2&1\\-4&5\end{bmatrix}=14.$$

Since $\vec{\mathbf{v}}_1=(2/7)\vec{\mathbf{w}}_1+(3/7)\vec{\mathbf{w}}_2$ and $\vec{\mathbf{v}}_2=-(1/7)\vec{\mathbf{w}}_1+(2/7)\vec{\mathbf{w}}_2$, the change of basis matrix is $\begin{bmatrix}2/7&-1/7\\3/7&2/7\end{bmatrix}$, with determinant 1/7.

(c) The determinant is 7, the inverse of the determinant in part (b). (This uses Theorem 4.8.11.) We can also do it by direct computation: Since $\vec{\mathbf{w}}_1 = 2\vec{\mathbf{v}}_1 - 3\vec{\mathbf{v}}_2$ and $\vec{\mathbf{w}}_2 = \vec{\mathbf{v}}_1 + 2\vec{\mathbf{v}}_2$, the change of basis matrix is $\begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$, with determinant +7. But this is unnecessary work.

If you find "new to old" description given in the text confusing, here is another description.

Let V be a vector space with two bases, basis $\{\mathbf{v}'\}$ and basis $\{\mathbf{v}\}$. You can then create two change of basis matrices. The ith column of the "$\{\mathbf{v}'\}$ to $\{\mathbf{v}\}$" change of basis matrix consists of the coefficients of the ith basis vector of $\{\mathbf{v}'\}$, written in terms of the basis vectors of $\{\mathbf{v}\}$: if $\mathbf{v}'_i = p_{1,i}\mathbf{v}_1 + \cdots + p_{n,i}\mathbf{v}_n$, the ith column of the change of basis matrix $[P_{\{\mathbf{v}'\},\{\mathbf{v}\}}]$ is $\begin{bmatrix} p_{1,i} \\ \vdots \\ p_{n,i} \end{bmatrix}$.

This matrix allows us to translate any vector "written" in basis $\{\mathbf{v}'\}$ into the same vector "written" in basis $\{\mathbf{v}\}$: if

$$a_1\mathbf{v}'_1 + \cdots + a_n\mathbf{v}'_n = b_1\mathbf{v}_1 + \cdots + b_n\mathbf{v}_n, \quad \text{then} \quad [P_{\{\mathbf{v}'\},\{\mathbf{v}\}}]\vec{\mathbf{a}} = \vec{\mathbf{b}}.$$

Thus we start by knowing how to write the basis vectors of $\{\mathbf{v}'\}$ in basis $\{\mathbf{v}\}$) and *we end up with the same knowledge* about any vector written in the basis $\{\mathbf{v}'\}$.

The "$\{\mathbf{v}\}$ to $\{\mathbf{v}'\}$" change of basis matrix is the inverse of the $\{\mathbf{v}'\}$ to $\{\mathbf{v}\}$" change of basis matrix. It consists of the coefficients of the ith basis vector of $\{\mathbf{v}\}$, written in terms of the basis vectors of $\{\mathbf{v}'\}$. This matrix will take the coefficients needed to write any vector in basis $\{\mathbf{v}\}$ and will give the coefficients needed to write that same vector in basis $\{\mathbf{v}'\}$.

The bottom line: If you are asked to write the change of basis matrix to go from basis A to basis B, the first thing you must do is to write each basis vector of A as a linear combination of the basis vectors of B. The coefficients you get are the entries of your matrix, but make sure to write them as a column, not a row: the coefficients used to write the ith basis vector of A in terms of the basis vectors of B form the ith *column* of the A to B change of matrix.

Page 156 Solution 6.3.9: We asked for a unit vector field, so we need to divide $\vec{F}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{bmatrix} -y \\ x-1 \end{bmatrix}$ by its length, which is $\sqrt{(x-1)^2 + y^2} = 2$.

Page 157 Solution 6.3.15: In the second paragraph, $F\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = x_1^2 + x_2^2 + x_3^2 = 0$ is nonsense, and the $2d_x$ in the derivative should be $2x_2$:

... The subset M is a 3-manifold in $\mathbb{R}^4$, since the fourth variable is a function of the other three. This 3-manifold is given by the equation $F(\mathbf{x}) = 0$, where $F\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = x_1^2 + x_2^2 + x_3^2 - x_4$. To find an orienting form using Proposition 6.3.8 we would compute the derivative $[2x_1\,,\,2x_2\,,\,2x_3\,,\,-1]$ to get the normal vector $\vec{\mathbf{n}} = \begin{bmatrix} 2x_1 \\ 2x_2 \\ 2x_3 \\ -1 \end{bmatrix}$, and compute ... .

Page 158 Parenthetical comment after the first displayed equation: this would probably be clearer as follows:

(Theorem 3.2.4: If $F(\mathbf{x}) = 0$ describes a manifold M , the tangent space $T_{\mathbf{x}}M$ is the kernel of the derivative of F.)

Page 161 Solution 6.5.7, first line: Figure 6.5.4, not 6.5.3.

Page 167 Solution 6.6.7, part (b): In the first displayed equation, the second function should be g, not f.

Page 172 Solution 6.7.11, part (b), end of first paragraph: Theorem 6.7.8, not A6.7.8.

Page 173 Solution 6.8.3 (c), sub-part (c): $\Phi_{\vec{F}}(\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2, \vec{\mathbf{v}}_3)$ is meaningful if it is in $\mathbb{R}^4$.

Page 173 Solution 6.8.5 is garbled. Here is the correct version.

6.8.5 (a) $\nabla f \begin{pmatrix} x \\ y \end{pmatrix} = \begin{bmatrix} 0 \\ 2y \end{bmatrix}$ (b) $\nabla f \begin{pmatrix} x \\ y \end{pmatrix} = \begin{bmatrix} 2x \\ -2y \end{bmatrix}$

(c) $\nabla f \begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{x^2+y^2} \begin{bmatrix} 2x \\ 2y \end{bmatrix}$ (d) $\nabla f \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \dfrac{\operatorname{sgn}(x+y+z)}{|x+y+z|} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

Page 173 Solution 6.8.7: The d after the third equal sign should be dropped:

$$df = W_{\operatorname{grad} f} = W_{\begin{bmatrix} F_1 \\ F_2 \end{bmatrix}} = (F_1\,dx + F_2\,dy)$$

Page 174 We had omitted part (c) of Solution 6.8.11. Below we compute, in great detail, curl and div of the first vector field, directly from the definition of the exterior derivative. This is an instructive exercise, if somewhat tedious; we recommend that you do at least one of the four.

Curl of the vector field in (a)

We will compute $dW_{\vec{F}} = \Phi_{\operatorname{curl} \vec{F}}$, the flux form field of $\operatorname{curl} \vec{F}$, and from that we will compute $\operatorname{curl} \vec{F}$. Since

$$W_{\vec{F}} = x^2y\,dx - 2yz\,dy + x^3y^2\,dz$$

is a 1-form, its exterior derivative is a 2-form. By Definition 6.7.1, we have

$$dW_{\vec{F}} = \lim_{h\to 0} \frac{1}{h^2} \int_{\partial(P_{\mathbf{x}}(h\vec{\mathbf{v}}_1, h\vec{\mathbf{v}}_2))} x^2y\,dx - 2yz\,dy + x^3y^2\,dz.$$

If you attempt to compute this using random vectors $\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2$, you will regret it. Instead, remember (Theorem 6.1.7) that any 2-form can be written in terms of the elementary 2-forms.

So

$$dW_{\vec{F}} = \Phi_{\text{curl}\,\vec{F}} = a\,dx \wedge dy + b\,dy \wedge dz + c\,dx \wedge dz$$

for some coefficients a, b, c. Theorem 6.1.7 says that to determine the coefficients, we should evaluate $dW_{\vec{F}}$ on the standard basis vectors.

Thus to determine the coefficient of $dx \wedge dy$ we will integrate $W_{\vec{F}}$ over the oriented boundary of the parallelogram spanned by $h\vec{\mathbf{e}}_1, h\vec{\mathbf{e}}_2$, computing

$$\lim_{h\to 0} \frac{1}{h^2} \int_{\partial P_{\mathbf{x}}(h\vec{\mathbf{e}}_1, h\vec{\mathbf{e}}_2)} x^2 y\,dx - 2yz\,dy + x^3 y^2\,dz.$$

To do this, we parametrize each edge of the parallelogram shown in the figure below and give it a plus or minus depending on its orientation:

(1)	$P_{\mathbf{x}}(h\vec{\mathbf{e}}_1)$	parametrized by	$\gamma_1(t) = \mathbf{x} + t\vec{\mathbf{e}}_1$
(2)	$P_{\mathbf{x}+h\vec{\mathbf{e}}_1}(h\vec{\mathbf{e}}_2)$	parametrized by	$\gamma_2(t) = \mathbf{x} + h\vec{\mathbf{e}}_1 + t\vec{\mathbf{e}}_2$
(3)	$P_{\mathbf{x}+h\vec{\mathbf{e}}_2}(h\vec{\mathbf{e}}_1)$	parametrized by	$\gamma_3(t) = \mathbf{x} + h\vec{\mathbf{e}}_2 + t\vec{\mathbf{e}}_1$
(4)	$P_{\mathbf{x}}(h\vec{\mathbf{e}}_2)$	parametrized by	$\gamma_4(t) = \mathbf{x} + t\vec{\mathbf{e}}_2$

First, we will determine the orientation of each edge. Using Proposition 6.6.15, we have

$$\partial P_{\mathbf{x}}(h\vec{\mathbf{e}}_1, h\vec{\mathbf{e}}_2) = (-1)^0 \big(P_{\mathbf{x}+h\vec{\mathbf{e}}_1}(h\vec{\mathbf{e}}_2) - P_{\mathbf{x}}(h\vec{\mathbf{e}}_2)\big) + (-1)^1 \big(P_{\mathbf{x}+h\vec{\mathbf{e}}_2}(h\vec{\mathbf{e}}_1) - P_{\mathbf{x}}(h\vec{\mathbf{e}}_1)\big),$$

so edges (1) and (2) come with a plus sign, while (3) and (4) get a minus.

Figure for solution 6.8.11, part c. The parallelogram spanned by $h\vec{\mathbf{e}}_1, h\vec{\mathbf{e}}_2$. In our calculations, we denote by x, y, z the entries of $\mathbf{x}$: $\mathbf{x} = \begin{pmatrix} x \\ y \\ x \end{pmatrix}$.

We must integrate $x^2 y\,dx - 2yz\,dy + x^3 y^2\,dz$ over each parametrized edge, but note that for the first edge (and the third), we are only concerned with $x^2 y\,dx$, since $dy(\vec{\mathbf{e}}_1) = 0$ and $dz(\vec{\mathbf{e}}_1) = 0$. If you don't see this, recall (Equation 6.4.38) how we integrate forms. To integrate $x^2 y\,dx - 2yz\,dy + x^3 y^2\,dz$ over the first edge, we integrate $x^2 y\,dx - 2yz\,dy + x^3 y^2\,dz(\overrightarrow{D_t \gamma_1}(t))$ over the first edge parametrized by γ_1. But $\overrightarrow{D_t \gamma_1}(t) = \vec{\mathbf{e}}_1$.

Thus for edge (1) we compute

$$\frac{1}{h^2} \int_{\partial(P_{\mathbf{x}}(h\vec{\mathbf{e}}_1, h\vec{\mathbf{e}}_2))} x^2 y\,dx = \frac{1}{h^2} \int_0^h \overbrace{(x+t)^2 y}^{x^2 y \text{ eval. at } \gamma_1(t)} \quad dx(\vec{\mathbf{e}}_1)\,dt$$

$$= \frac{1}{h^2} \int_0^h x^2 y + 2xyt + t^2 y\,dt = \frac{1}{h^2}\left(x^2 y h + xyh^2 + \dots\right).$$

Notice that t^2y will give a term in h^3; once we divide by h^2 it will be a term in h, which will go to 0 as $h \to 0$. So we can ignore it. The terms that count for edge (1) are $\frac{x^2y}{h} + xy$, both taken with a +. (The h in the denominator might be worrisome — what will happen as $h \to 0$? But we will see that it cancels with a term from another edge. That is the point of having an oriented boundary!)

For edge (2) we are only concerned with $-2yz\,dy$, since $dx(\vec{\mathbf{e}}_2) = 0$ and $dz(\vec{\mathbf{e}}_2) = 0$. We compute

$$\frac{1}{h^2}\int_0^h -2yz - 2tz\,dt,$$

which gives $\frac{-2yz}{h} - z$.

A similar computation for edge (3) gives the terms $\frac{x^2y}{h} + xy + x^2$, each term taken with a minus sign; for edge (4) we get $\frac{-2yz}{h} - z$, also taken with a minus sign. Thus we have

$$\partial\big(P_{\mathbf{x}}(h\vec{\mathbf{e}}_1, h\vec{\mathbf{e}}_2)\big) = \underbrace{\frac{x^2y}{h} + xy}_{\text{from edge (1)}} + \underbrace{\frac{-2yz}{h} - z}_{\text{from edge (2)}} \underbrace{-\frac{x^2y}{h} - xy - x^2}_{\text{from edge (3)}} + \frac{2yz}{h} + z = -x^2.$$

We can substitute this for a in Equation (1):

$$\Phi_{\operatorname{curl}\vec{F}} = -x^2\,dx \wedge dy + b\,dy \wedge dz + c\,dx \wedge dz.$$

Recall (line after Definition 6.5.2) that $\Phi_{\vec{F}} = F_1\,dy \wedge dz - F_2\,dx \wedge dz + F_3\,dx \wedge dy$, so $-x^2$ should be the third entry of $\operatorname{curl}\vec{F}$, which is indeed what we got in part (a) (with considerably less effort!).

For the coefficient of $dx \wedge dz$ we integrate over the boundary of the parallelogram spanned by $h\vec{\mathbf{e}}_1, h\vec{\mathbf{e}}_3$. To save work, we note that for edges 1 and 3 we are interested only in $x^2y\,dx$, and for edges 2 and 4 we are only interested in $x^3y^2\,dz$. We get

$$\partial\big(P_{\mathbf{x}}(h\vec{\mathbf{e}}_1, h\vec{\mathbf{e}}_3)\big) = \underbrace{\frac{x^2y}{h} + xy}_{\text{from edge (1)}} + \underbrace{\frac{x^3y^2}{h} + 3x^2y^2}_{\text{from edge (2)}} \underbrace{-\frac{x^2y}{h} - xy}_{\text{from edge (3)}} - \frac{x^3y^2}{h} = 3x^2y^2$$

Since in the formula for $\Phi_{\vec{F}}$ the coefficient of $dx \wedge dz$ is $-F_2$, the second entry of $\operatorname{curl}\vec{F}$ should be $-3x^2y^2$, which is what we got in part (a).

A similar computation involving $\vec{\mathbf{e}}_2, \vec{\mathbf{e}}_3$ gives the coefficient for $dy \wedge dz$, i.e., the first entry of $\operatorname{curl}\vec{F}$.

Div of the vector field in (a)

Now let's compute the divergence of the vector field in (a), by computing $d\Phi_{\vec{F}} = \rho_{\operatorname{div}\vec{F}}$. Since

$$\Phi_{\vec{F}} = x^2y\,dy \wedge dz + 2yz\,dx \wedge dz + x^3y^2\,dx \wedge dy$$

is a 2-form, $d\Phi_{\vec{F}}$ is a 3-form and can be written $\alpha\,dx \wedge dy \wedge dz$ for some coefficient α. We will find α by integrating $x^2y\,dy \wedge dz + 2yz\,dx \wedge dz + x^3y^2\,dx \wedge dy$ over the oriented boundary of the parallelogram spanned by $h\vec{\mathbf{e}}_1, h\vec{\mathbf{e}}_2, h\vec{\mathbf{e}}_3$, shown below.

The boundary consists of six faces, which we parametrize as follows, where $0 \le s, t \le h$, and $\mathbf{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$; we use Proposition 6.6.15 to determine which faces are taken with a plus sign and which are taken with a minus sign:

1. $\gamma\begin{pmatrix} s \\ t \end{pmatrix} = \mathbf{x} + s\vec{\mathbf{e}}_1 + t\vec{\mathbf{e}}_2$, minus. (This is the base of the box shown above.)
2. $\gamma\begin{pmatrix} s \\ t \end{pmatrix} = \mathbf{x} + h\vec{\mathbf{e}}_3 + s\vec{\mathbf{e}}_1 + t\vec{\mathbf{e}}_2$, plus. (This is the top of the box.)
3. $\gamma\begin{pmatrix} s \\ t \end{pmatrix} = \mathbf{x} + s\vec{\mathbf{e}}_1 + t\vec{\mathbf{e}}_3$, plus. (This is the front of the box.)
4. $\gamma\begin{pmatrix} s \\ t \end{pmatrix} = \mathbf{x} + h\vec{\mathbf{e}}_2 + s\vec{\mathbf{e}}_1 + t\vec{\mathbf{e}}_3$, minus. (This is the back of the box.)
5. $\gamma\begin{pmatrix} s \\ t \end{pmatrix} = \mathbf{x} + s\vec{\mathbf{e}}_2 + t\vec{\mathbf{e}}_3$, minus. (This is the left side of the box.)
6. $\gamma\begin{pmatrix} s \\ t \end{pmatrix} = \mathbf{x} + h\vec{\mathbf{e}}_1 + s\vec{\mathbf{e}}_2 + t\vec{\mathbf{e}}_3$, plus. (This is the right side of the box.)

For the integral over the first face, we are only concerned with $x^3y^2\,dx \wedge dy$, since $dy \wedge dz(\vec{\mathbf{e}}_1, \vec{\mathbf{e}}_2) = 0$ and $dx \wedge dz(\vec{\mathbf{e}}_1, \vec{\mathbf{e}}_2) = 0$. So we have

$$\frac{1}{h^3}\int_{\partial P_{\mathbf{x}}(h\vec{\mathbf{e}}_1, h\vec{\mathbf{e}}_2)} x^3y^2\,dx \wedge dy = \frac{1}{h^3}\int_0^h \int_0^h \overbrace{(x+s)^3(y+t)^2 dx \wedge dy}^{x^3y^2 dx\wedge dy \text{ eval. at } \gamma\begin{pmatrix} s \\ t \end{pmatrix}} \overbrace{(\vec{\mathbf{e}}_1, \vec{\mathbf{e}}_2)}^{\overrightarrow{D_s\gamma}, \overrightarrow{D_t\gamma}}\,ds\,dt$$

After discarding everything that will give terms in h^4, dividing by h^3 and giving the correct orientation, we are left with the following for the first face:

$$\frac{-x^3y^2}{h} - \frac{3x^2y^2}{2} - x^3y.$$

For the top of the box we get exactly the same terms, but with opposite sign.

For the third face (the front), we get

$$\frac{2yz}{h} + y.$$

For the fourth face (the back), we get

$$\frac{-2yz}{h} - y - 2z.$$

For the fifth and sixth faces, we are concerned only with $x^2y\,dy \wedge dz$. The fifth face gives

$$\frac{-x^2y}{h} - \frac{x^2}{2}.$$

For the sixth face, we get

$$\frac{x^2y}{h} + 2xy + \frac{x^2}{2}.$$

After cancellations, this leaves $2xy - 2z$. Thus

$$d\Phi_{\vec{F}} = \rho_{\operatorname{div}\vec{F}} = (2xy - 2z)\, dx \wedge dy \wedge dz.$$

By Equation 6.5.11, $\operatorname{div}\vec{F} = 2xy - 2z$, which is what we got in part (a).

We will not work out curl and div of the second vector field, since the procedure is the same and you can check your results from part (b).

Page 176 The end of Solution 6.9.5 should be

So

$$\operatorname{vol}_3(A) = abc\left(\frac{1}{8}\right)\left(\frac{4\pi}{3}\right) = \frac{abc\,\pi}{6}.$$

Finally, $\displaystyle\int_S \omega = 3\operatorname{vol}_3(A) = \frac{abc\,\pi}{2}.$

Page 177 Solution 6.9.7: In the last three lines of the four-line equation, we omitted $|d^2\mathbf{u}|$.

Page 178 Solution 6.10.11: the function should be in parentheses:

$$\int_{\text{ball}} d\Phi_{\begin{bmatrix} x^2 \\ y^2 \\ z^2 \end{bmatrix}} = \int_{\text{ball}} (2x + 2y + 2z)\, dx \wedge dy \wedge dz$$

Page 179 Grammatical typo in Solution 6.10.15, part (b): there should be end quotes after "the surface $X_{p,q}$ of equation $z_1^p + z_2^q$."

Page 181 We included extra solutions by accident.

Page 182 Last displayed equation of Solution 6.11.5 $\vec{\nabla}$, not d:

$$\begin{bmatrix} \dfrac{x}{x^2+y^2} \\ \dfrac{y}{x^2+y^2} \end{bmatrix} = \vec{\nabla}\left(\frac{1}{2}\ln(x^2+y^2)\right).$$

Page 187 Solution 6.19 is correct but does not use Theorem 6.8.3. Here is a solution that does:

We have $\operatorname{curl}(\operatorname{grad} f) = 0$ since

$$0 = ddf = dW_{\vec{\nabla} f} = \Phi_{\nabla \times \vec{\nabla} f} = \Phi_{\operatorname{curl}(\operatorname{grad} f)}.$$

We have $\operatorname{div}\operatorname{curl}\vec{F} = 0$ since

$$0 = ddW_{\vec{F}} = d\Phi_{\operatorname{curl}\vec{F}} = \rho_{\operatorname{div}\operatorname{curl}\vec{F}} = (\operatorname{div}\operatorname{curl}\vec{F})\, dx \wedge dy \wedge dz.$$

Page 196 The book contains no solution for Exercise A14.1. Here it is:

A14.1 By definition, $H = (1/2)(A_{2,0} + A_{0,2})$. The calculations in the proof of Proposition 3.8.9 give us expressions for $A_{2,0}$ and $A_{0,2}$. It is now straightforward to show that H is given by the desired expression. Remember we have set $c = \sqrt{a_1^2 + a_2^2}$, so that $a_1^2 + a_2^2 = c^2$.

$$\begin{aligned}
H &= \frac{1}{2}\left(A_{2,0} + A_{0,2}\right) \\
&= -\frac{1}{2c^2\sqrt{1+c^2}}\left(a_{2,0}a_2^2 - 2a_{1,1}a_1a_2 + a_{0,2}a_1^2\right) - \frac{1}{2c^2(1+c^2)^{3/2}}\left(a_{2,0}a_1^2 + 2a_{1,1}a_1a_2 + a_{0,2}a_2^2\right) \\
&= -\frac{1}{2c^2\left(1+c^2\right)^{3/2}}\left(a_{2,0}\left(a_2^2(1+c^2) + a_1^2\right) - 2a_{1,1}a_1a_2\left(1 + c^2 - 1\right) + a_{0,2}\left(a_1^2(1+c^2) + a_2^2\right)\right) \\
&= -\frac{1}{2c^2\left(1+c^2\right)^{3/2}}\left(a_{2,0}c^2(1+a_2^2) - 2a_{1,1}a_1a_2c^2 + a_{0,2}c^2(1+a_1^2)\right) \\
&= -\frac{1}{2\left(1+c^2\right)^{3/2}}\left(a_{2,0}(1+a_2^2) - 2a_{1,1}a_1a_2 + a_{0,2}(1+a_1^2)\right)
\end{aligned}$$