Vector Calculus, Linear Algebra, and Differential Forms: A Unified Approach

More notes and errata for the 3rd edition

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Note: Mathematical typos and errors in the text proper are marked with two stars; mathematical typos and errors in the exercises are marked with one star.

We omit from this list minor notational inconsistencies that should cause no confusion (for example, sometimes we write $\text{vol}_n A$ and sometimes $\text{vol}_n(A)$, with parentheses).

Many thanks to

David Besser, Tara Holm, Thomas Madden, Lewis Robinson,

Leonard Smiley, Leo Trottier, and Charles Yu

for their contributions to this list

Page 14 Example 0.4.8: In the second paragraph, 1st and 2nd lines, replace "positive" by "nonnegative".

Page 18 6th line from bottom: "since $[x]_j$ is one of them", not "since $[a]_j$ is one of them". 3rd line from bottom: $j-1$ not $j+1$, in two places. Next 2 lines: b_{j-1}, not b_{j+1}, in three places.

Page 19 First line: b_{j-2} and b_{j-3}, not b_{j+2} and b_{j+3}.

Page 19 In example 0.5.6, 2.020202 should be 2.020202...:

For example, $2.020202\ldots = 2 + 2(.01) + 2(.01)^2 + \cdots$.

Page 21 Exercise 0.5.3: We are assuming that $a \le b$. By convention, $[a, b]$ implies $a \le b$.

Page 22 First line after equation 0.6.2, replace "formed by the elements of the diagonal digits" by "formed by the diagonal digits." In the last line of the same paragraph, replace "same nth decimal" by "same nth decimal digit".

Replace the first sentence in the last paragraph before the heading "Existence of transcendental numbers" by

Infinite sets that can be put in one-to-one correspondence with the natural numbers are called *countable* or *countably infinite*.

Page 23 Figure 0.6.3: It seems that we misattributed the statement, "This isn't math, it's theology" to Hermite. Leonard Smiley informs us that this quotation is usually associated with Paul Gordan commenting on the Hilbert basis theorem. In an online forum, Walter Felscher of the University of Tuebingen writes that the saying is generally attributed to Gordan, but that "it does not seem to appear in Gordan's writings, and so it has the status of hearsay."

PAGE 30 Line 16: " ... and in the complex case ... "

**PAGE 36 In figure 1.1.3, the point labeled $\mathbf{a}$ should be $\mathbf{b}$ and the point labeled $\mathbf{b}$ should be $\mathbf{a}$. The existing figure shows $\overrightarrow{\mathbf{b}-\mathbf{a}}$ not $\overrightarrow{\mathbf{a}-\mathbf{b}}$.

PAGE 42 First margin note: Lewis Robinson says he uses the mnemonic "roman catholic" for "row then column".

PAGE 57 Footnote: Both affine and linear transformations give as their output first-degree polynomials in their inputs. Linear transformations are affine, of course.

PAGE 58 End of margin note: " ... and then multiplying ... ", not "the multiplying".

PAGE 65 In the 4th line of the footnote, exercise 0.6.7, not 0.6.8.

PAGE 92 3 lines before equation 1.5.26: replace "we can define the limit $\lim_{\mathbf{x}\to\mathbf{x}_0} f(\mathbf{x})$" by

"it makes sense to ask whether the limit $\lim_{\mathbf{x}\to\mathbf{x}_0} f(\mathbf{x})$ exists."

PAGE 97 Definition 1.5.26, last line: The second f should be bold: $|\mathbf{f}(\mathbf{x}) - \mathbf{f}(\mathbf{x}_0)| < \epsilon$, not $|\mathbf{f}(\mathbf{x}) - f(\mathbf{x}_0)| < \epsilon$.

**PAGE 98 In the 5th line, the first $\mathbf{x}$ should be $\mathbf{x}_n$: "there exists N such that when $n > N$ we have $|\mathbf{f}(\mathbf{x}_n) - \mathbf{f}(\mathbf{x}_0)| < \epsilon$."

PAGE 99 Line 3: The codomain of a monomial function is $\mathbb{R}$.

PAGE 102 Line 15: "This tells us that $\sum_k |A|^k$ converges", not "This tells us what ... ".

**PAGE 108 Caption for figure 1.6.2: If the fractional part of a number α is between 0 and 1/2, then $\sin 2\pi\alpha \geq 0$; if it is between 1/2 and 1, then $\sin 2\pi\alpha \leq 0$. (We are changing from strict inequalities because "between 0 and 1/2" is ambiguous.)

PAGE 108 Top of the page: Replace the last paragraph of the proof of theorem 1.6.3 by

"Consider $\mathbf{a} \in \mathbb{R}^n$ such that the mth decimal digit of each coordinate a_k agrees with the mth decimal digit of the kth coordinate of $\mathbf{x}_{i(m)}$. Then $\left|\mathbf{x}_{i(m)} - \mathbf{a}\right| < n10^{-m}$ for all m. Thus $\mathbf{x}_{i(m)}$ converges to $\mathbf{a}$. Since C is closed, $\mathbf{a}$ is in C."

PAGE 109 It is also widely believed that $\frac{1}{2\pi}$ is normal.

PAGE 109 Paragraph after example 1.6.4: Perhaps we should replace "intuitionists" by "constructivists".

PAGE 114 In the last displayed equation in the margin, the $-a$ in the first line should be $-b$: $|a| = |a + b - b\,|$

**PAGE 117 2 lines before equation 1.6.26, replace "$|u| = \rho < 1$" by "$|u| = \rho \leq 1$". Line after equation 1.6.26, replace "$|b_j u^j| \geq$" by "$|b_j u^j| >$, in two places.

PAGE 128 The three lines before equation 1.7.24 should be replaced by "Equation 1.7.20 says nothing about the direction in which $\vec{\mathbf{h}}$ approaches $\mathbf{0}$; it can approach $\mathbf{0}$ from any direction. In particular, we can set $\vec{\mathbf{h}} = t\vec{\mathbf{e}}_i$, and let the number t tend to 0, giving"

In equations 1.7.24 and 1.7.25, $f(\mathbf{a})$ should be $\mathbf{f}(\mathbf{a})$.

**PAGE 129 Proof of proposition 1.7.11, next to last line: "$\lim_{\vec{\mathbf{h}}\to\mathbf{0}}(\mathbf{f}(\mathbf{a}+\vec{\mathbf{h}})-\mathbf{f}(\mathbf{a}))$ must also be $\mathbf{0}$."

PAGE 141 Equation 1.8.6: The 0 in $0\vec{\mathbf{h}}$ is a zero matrix, and might be written [0]. The next two 0's are vectors, and should be bold.

PAGE 144 There should be no overset arrow in the displayed equation in the margin; $D_j(\mathbf{f}\circ\mathbf{g})_i(\mathbf{a})$ is not a vector.

*PAGE 147 Exercise 1.8.10: The beginning of part a should read

"Show that if a function $f : \mathbb{R}^2 \to \mathbb{R}$ can be written $\varphi(x^2 + y^2)$ for some differentiable function $\varphi : \mathbb{R} \to \mathbb{R}$, then ... "

*PAGE 159 Exercise 1.34 contains two errors. First, "such that $h(\mathbf{x}) = -h(\mathbf{x})$." should have been "such that $h(-\mathbf{x}) = -h(\mathbf{x})$." Second, it is not true that under the given hypotheses, "f has directional derivatives in all directions, at all points of $\mathbb{R}^2$." This exercise should be replaced by the following:

1.34 Let $h:\mathbb{R}\to\mathbb{R}$ be a C^1 function, periodic of period 2π, and define $f:\mathbb{R}^2\to\mathbb{R}$ by

$$f\begin{pmatrix} r\cos\theta \\ r\sin\theta \end{pmatrix} = rh(\theta).$$

a. Show that f is a continuous real-valued function on $\mathbb{R}^2$.

b. Show that f is differentiable on $\mathbb{R}^2 - \{\mathbf{0}\}$.

c. Show that all directional derivatives of f exist at $\mathbf{0}$ if and only if $h(\theta) = -h(\theta+\pi)$ for all θ.

d. Show that f is differentiable at $\mathbf{0}$ if and only if $h(\theta) = a\cos\theta + b\sin\theta$ for some numbers a and b.

PAGE 171 Two lines before equation 2.2.9, a comma is needed: "Denote these nonzero entries by $\tilde{a}_1, \dots, \tilde{a}_k$," not "Denote ... by $\tilde{a}_1, \dots \tilde{a}_k$."

**PAGE 171 On the left side of equation 2.2.9, x_i, not x_1.

PAGE 216 Next to last line, "how useful it can be to change bases", not "how useful it can to change bases ".

PAGE 241 In the second margin note, absolute value of x_i: $|x_1| \le \sqrt{x_1^2 + x_2^2}$.

*PAGE 277 Exercise 2.10.16: The same letter A is used to denote both a specific matrix and a general matrix A in the neighborhood U. This should be:

2.10.16 The matrix $A_0 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ satisfies $A_0^3 = I$.

True or false? There exists a neighborhood $U \subset \text{Mat}\,(3,3)$ of I and a continuously differentiable function $g : U \to \text{Mat}\,(3,3)$ with $g(I) = A_0$ and $(g(A))^3 = A$ for all $A \in U$ (i.e., $g(A)$ is a cube root of A).

**PAGE 307 In equations 3.2.1 and 3.2.2, $[\mathbf{Df}(\mathbf{c})]$ should be $[\mathbf{Df}(\mathbf{b})]$.

PAGE 364 First margin note, line 7: "Lagrange", not "Langrange".

PAGE 496 Margin note: In the line immediately before the margin paragraph starting "It is surprising," *measure thory* should be *measure theory*.

PAGE 553 5 lines before the heading "Elementary forms": "the parallelogram spanned by those k vectors" should be "the k-parallelogram spanned … ".

PAGE 571 Definition 6.3.1, part 1, lines 3–4: "Two forms define the same orientation" (not "same direction").

**PAGE 620 Remark 6.7.3: At the end of the first paragraph, some subscripts are wrong. This should be

"the integral over a face should be something of the form $h^k(a_0 + a_1 h + a_2 h^2 \dots)$; it would appear that dividing by h^{k+1} would give a term a_0/h … ".

PAGE 672 Exercise 6.31: In the line before the third displayed equation, "3-dimensional", not "3-dimensonal".

PAGE 727 Next to last margin note: "denominator in", not 'denominatorin".

PAGE 766 6th line after equation A23.1: "the remainder is $|R(f)(\vec{\mathbf{x}})|$," not "the remainder is $|R(\vec{\mathbf{x}})|$."

PAGE 770 In proposition A24.2, the line before equation A24.2 should be

" … and by $y_1, \dots, y_m$ the coordinates in $\mathbb{R}^m$. Then"

PAGE 771 In example A24.5 we work from the definition. As Leonard Smiley suggests, it would have been good to also give a more computational approach. Using the rules

$$\mathbf{f}^*(\varphi_1 \wedge \varphi_2) = \mathbf{f}^*\varphi_1 \wedge \mathbf{f}^*\varphi_2 \quad \text{and} \quad \mathbf{f}^*(d\varphi) = d\mathbf{f}^*\varphi,$$

we can compute:

$$\begin{aligned} \mathbf{f}^*(y_2 dy_1 \wedge dy_3) &= (x_1 x_2)\big(d(x_1^2) \wedge d(x_2^2)\big) \\ &= (x_1 x_2)(2x_1 dx_1) \wedge (2x_2 dx_2) \\ &= 4x_1^2 x_2^2 \, dx_1 \wedge dx_2. \end{aligned}$$