Vector Calculus, Linear Algebra, and Differential Forms: A Unified Approach, 4th Edition

New Erratum

March 28, 2011

Our thanks to Skylar Sutherland for alerting us to this error.

Page 137 In equations 1.7.59 and 1.7.60 we wrote 1 instead of the identity matrix I, but the length of I is not 1. The part of the proof that is on this page should be replaced by the following:

difference between the increment of the function and its approximation:

$$\underbrace{\big((A+H)^{-1}-A^{-1}\big)}_{\text{increment to function}} - \underbrace{(-A^{-1}HA^{-1})}_{\substack{\text{linear function of}\\ \text{increment to variable}}}$$

$$= \Big((-A^{-1}H)^2 + (-A^{-1}H)^3 + \cdots\Big)A^{-1} \qquad 1.7.59$$

$$= (-A^{-1}H)\Big((-A^{-1}H) + (-A^{-1}H)^2 + (-A^{-1}H)^3 + \dots\Big)A^{-1}.$$

Proposition 1.4.11 and the triangle inequality (theorem 1.4.9) give

$$\begin{aligned}
&\big|(A+H)^{-1} - A^{-1} + A^{-1}HA^{-1}\big| \\
&\quad\underbrace{\le}_{\text{prop. 1.4.11}} |A^{-1}H|\,\Big|(-A^{-1}H) + (-A^{-1}H)^2 + (-A^{-1}H)^3 + \dots\Big||A^{-1}| \\
&\quad\underbrace{\le}_{\text{triangle ineq.}} |A^{-1}H||A^{-1}|\big(\overbrace{|A^{-1}|\,|H| + |A^{-1}|^2\,|H|^2 + \cdots}^{\text{convergent geometric series}}\big) \\
&\quad\le |A^{-1}|^2|H|\,\frac{|A^{-1}||H|}{1-|A^{-1}||H|}
\end{aligned} \qquad 1.7.60$$

Now suppose H so small that $|A^{-1}|\,|H| < 1/2$, so that

$$\frac{1}{1-|A^{-1}|\,|H|} \le 2. \qquad 1.7.61$$

We see that

$$\lim_{H\to 0}\frac{1}{|H|}\big|(A+H)^{-1} - A^{-1} + A^{-1}HA^{-1}\big| \;\le\; \lim_{H\to 0} 2|H||A^{-1}|^3 = [0]. \qquad \square$$