Functional Analysis
Volume 1: A Gentle Introduction

Notes and errata, as of May 21, 2018

The posting marked in red are new.

We thank Richard Palas for his contributions to this list.

Page 3 Line before inequality (0.15): apply Inequality (0.11), not 0.3.

Page 8 In Example 1.1.6, reference is made to Appendix B.5; it should be to Notation B.2.2 (in Appendix B.2).

Page 49 Exercise 1.6.2: The second sentence should be

Show that if the closure of M in N is M and the closure of N in X is N, then the closure of M in X is also M.

Page 69 line after inequality (1.8.7): "This shows that $\mathbf{t} \in l^p$. Finally, (1.8.4) becomes"

should be replaced by

"This shows that $\mathbf{t} \in l^\infty$. So inequality (1.8.4) becomes"

Page 92 Three lines before inequalities (1.11.15): $x \in E$ should be $z \in E$.

Page 100 Exercise 1.11.1 repeats Corollary 1.11.12. When the book is reprinted we will replace it by:

Give an example of a continuous bijection from a compact topological space X into a topological space Y that is not a homeomorphism.

Page 151 The last line of Definition 2.5.16 should be

neighborhood U of $\mathbf{0}$, there is some $B \in \mathcal{B}$ such that $B \subset U$.

Page 155 Next-to-last line of the proof of Corollary 2.5.26: "...such that $rB \subset W$" should be

"...such that $rB \subset U$."

Page 155 Exercise 2.5.3 was wrong (A sum of two sets, at least one of which is closed, need not be closed). It should be replaced by

Let U and V be subsets of a normed space. Show that $\overline{U + V} = \overline{U} + \overline{V}$.

Page 156 Exercise 2.5.16: In the definition of M, $c_1 v_1$, not $v_1 v_1$

Page 161 Three lines before Theorem 2.6.5: "a subset A", not "a subset M".

Page 162 Exercise 2.6.5: In part 3, $\{x \in X | \, p(x) < 1\}$ should be

$$\{x \in X | \, p(x) < \epsilon\}.$$

In the last sentence of the exercise, "on X if and only if" should be "on V if and only if".

Page 193 Near the end of the proof of Proposition 2.10.15: CorollaryB.3.5 should be Corollary B.3.5.

Page 198 Exercise 2.10.18: $\dim c/c_0 = 1$, not $\dim c/c_0 = \infty$.

Page 199 There should be a period after equation (2.11.4).

Page 224 line 5: $\mathbb{R}^n$ not R^n

Page 227 Exercise 3.2.7, part 3: $f = g + Tf$ should be $f = g + T_c f$.

Page p. 244 There should be an end of proof symbol at the end of the sentence immediately before Example 3.4.2.

Page p. 261 Definition 3.5.12: There should be no period at the end of equation (3.5.21); there should be a period at the end of the next line.

Page 351 Theorem 4.6.17 should be in a grey box.

Page 371 last line, "from the Corollary 5.1.12" should be "from Corollary 5.1.12".

Page 408 The line immediately before Example 5.4.17 should be "is maximal if $\dim \mathcal{A}/I = 1$" (not if and only if).

Page 454 5-6 lines before (6.3.54) is there a bad line break.

Page 466 In Example A.2.7, the first sentence should be: "Let S, $P(S)$, and $\preceq$ be as in part 2 of Examples A.2.5". (The problem with the existing text is that S has to be defined as $\{a, b\}$, which it is in Examples A.2.5, but not in Examples A.2.3.)

Page 486 Theorem B.4.2. and Definition B..4.3 should be in grey boxes.

Page 495, solution to Exercise 1.1.11, at the end of the third line:

$$|x_k|^p \leq |x_k|^q \quad \text{should have been} \quad |x_k|^q \leq |x_k|^p.$$

Page 515 The solution to Exercise 1.13.15 is missing. Here it is:

False. Let X be any countable discrete topological space. Then, by Exercise 1.13.1, X is fat in itself.

Page 523 Solution 2.5.3 should be replaced by

For each $r > 0$, let B_r denote the open ball of radius r centered at $\mathbf{0}$. Let $\mathcal{B} := B(\mathbf{0}, r)r > 0$. Then $\mathcal{B}$ is a local base for X. Note also that $B_{\frac{r}{2}} + B_{\frac{r}{2}} \subset B_r$. Let $x \in \overline{U+V}$. Then by Example 2.5.18, $x \in (U+V) + B_r$ for each $r > 0$. Thus, for all $r > 0$,

$$x \in (U + B_{\frac{r}{2}}) + (V + B_{\frac{r}{2}}).$$

Hence,

$$x \in \bigcap_{r>0}(U + B_r) + \bigcap_{r>0}(V + B_r).$$

By Example 2.5.18 again, $\bigcap_{r>0}(U + B_r) = \overline{U}$ and $\bigcap_{r>0}(V + B_r) = \overline{V}$. Hence, $x \in \overline{U} + \overline{V}$. Therefore, $\overline{U+V} \subset \overline{U} + \overline{V}$. The reverse inclusion is already proved in formula (2.5.18).

Page 562 Exercise 3.5.7: $\mathcal{BC}(X, Y) \neq \emptyset$, not $\mathcal{BC}(X, y) \neq \emptyset$ and $\|S_n - T_0\| \to 0$, not $\|S_n - T\| \to 0$.

Page 573 Solution 3.9.1: The 5 in the displayed equation following "Hence"should be deleted.

Page 621 The two books listed in reference [57] should have been listed separately.

Page 626 "$C(X)$": add page 96

Page 627 "compact operator": add page 265 (Exercise 3.5.7)

Page 627 "continuity": add page 367

Page 629 "eigenvalue": add page 254 (Exercise 3.4.5)

Page 631 "Hilbert space, $\mathbb{R}^n$": add page 288

Page 639 "uniform boundedness principles": change 245 to 244