Teichmüller Theory and Applications

to Geometry, Topology, and Dynamics

Volume 1 Teichmüller Theory

Notes and Errata, posted Jan.4, 2008

Note: Mathematical errors are marked with a star.

We thank Mohan Ramachandran and Thomas Schmidt for their contributions to this list.

Page 14 Professor Mohan Ramachandran has pointed out that the argument of equation 1.4.8 is not quite clear: are we working in De Rham cohomology or singular (or Čech) cohomology. In singular or Čech cohomology, the argument is correct, but in de Rham cohomology, which I have been using mainly in this chapter (in equations 1.4.6 and 1.4.7) this requires a bit of further argument, since ρ_n is not smooth and doesn't obviously induce anything on de Rham cohomology spaces.

There are various ways around this. One is not to use de Rham cohomology at all: replace equations 1.4.6 and 1.4.7 by saying that the Poincaré dual of δ (i.e., intersection with δ) is a nonzero singular cohomology class, since δ intersects γ_1 transversely in a single point.

Another is to invoke de Rham's theorem, which is proved in Appendix A7.5. That seems a little heavy-handed, a clear case of using a sledge hammer to kill a fly.

Another is to show that continuous maps between smooth manifolds do induce homomorphisms on de Rham cohomology. This is of course true by de Rham's theorem, but can be proved much more easily, by approximation (at least on σ-compact manifolds), and using partitions of unity.

(1) First show that on any σ-compact manifold there exist Riemannian metrics that are *controlled at infinity* in the sense that there exists $\rho_0 > 0$ such that any pair of points distance $< \rho_0$ apart are joined by a unique geodesic.

(2) Next show that if X and Y are σ-compact manifolds with Riemannian metrics, then every continuous $f : X \to Y$ can be uniformly approximated by C^∞-maps, and if the metric of Y is controlled at infinity, then any two approximations within $\rho_0/2$ are smoothly homotopic.

Page 16 In the last paragraph before section 1.7, "Thus our map ... " would be better as "Thus, by the reflection principle, our map ... ."

(The reflection principle says that if $U \subset \mathbb{H}$ is open and $f : U \to \mathbb{C}$ is an analytic function such that $\operatorname{Im} f(z) \to 0$ when $\operatorname{Im} z \to 0$, then $f(z) = \overline{f(\overline{z})}$ extends f analytically to $U \cup U^* \cup (\overline{U} \cap \mathbb{R})$. We do not have to assume that f extends continuously to $\overline{U} \cap \mathbb{R}$.)

Page 214 Perhaps I should have elaborated on the last sentence before Figure 5.3.7:

... for each critical point of q' in Y and each critical trajectory emanating from it, mark the first intersection of that trajectory with J (if it exists), as illustrated in Figure 5.3.7. (We will see right after equation 5.3.12 that it does exist.)

***Page 397** The proof for proposition A7.5.6 is incorrect. Here is a corrected version:

PROOF Choose an exhaustion $V_1 \subset V_2 \cdots \subset U$ of U by open sets such that each V_i is relatively compact in V_{i+1}. Let U_i be the union of V_i and all the compact components of $U - V_i$; then every component of $\mathbb{C} - U_i$ contains points that are not in U. Further, choose C^∞ functions h_i on $\mathbb{C}$ that are identically 1 in U_i and identically 0 on $\mathbb{C} - U_{i+1}$.

Given $\alpha \in A^{0,1}(U)$, by equation A7.5.9 we can find $\beta_n \in A^{0,0}(U)$ such that $\overline{\partial}\beta_n = h_n\alpha$. Since we can write

$$\beta_n \stackrel{\text{def}}{=} \beta_0 + (\beta_1 - \beta_0) + \cdots + (\beta_n - \beta_{n-1}), \qquad A7.5.13$$

it is tempting to set $\beta = \beta_0 + \sum_{k=1}^{\infty}(\beta_{k+1} - \beta_k)$; unfortunately, the series does not converge. Instead, note that $\beta_{k+1} - \beta_k$ is analytic on a neighborhood of U_k, and every component of $\mathbb{C} - U_k$ contains points not in U. Thus by the Runge approximation theorem, there exist rational functions p_k analytic in U such that

$$\sup_{z \in \overline{U}_k} |\beta_{k+1}(z) - \beta_k(z) - p_k(z)| \le \frac{1}{2^k}. \qquad A7.5.14$$

Now the series

$$\beta \stackrel{\text{def}}{=} \beta_0 + (\beta_1 - \beta_0 - p_0) + \beta_2 - \beta_1 - p_1) + \cdots \qquad A7.5.15$$

converges uniformly on compact subsets of U, and β satisfies $\overline{\partial}\beta = \alpha$. $\square$