Teichmüller Theory and Applications

to Geometry, Topology, and Dynamics

Volume 1: Teichmüller Theory

We thank Laurent Bartholdi, Hendrik Chaltin, Sa'ar Hersonsky, Andrew Marshall, Curt McMullen, Mohan Ramachandran, and Thomas Schmidt for their contributions to this list.

Errata and notes: complete as of October 24, 2012

Page 14 Professor Mohan Ramachandran has pointed out that the argument of equation 1.4.8 is not quite clear: are we working in De Rham cohomology or singular (or Čech) cohomology. In singular or Čech cohomology, the argument is correct, but in de Rham cohomology, which I have been using mainly in this chapter (in equations 1.4.6 and 1.4.7) this requires a bit of further argument, since ρ_n is not smooth and doesn't obviously induce anything on de Rham cohomology spaces.

There are various ways around this. One is not to use de Rham cohomology at all: replace equations 1.4.6 and 1.4.7 by saying that the Poincaré dual of δ (i.e., intersection with δ) is a nonzero singular cohomology class, since δ intersects γ_1 transversely in a single point.

Another is to invoke de Rham's theorem, which is proved in Appendix A7.5. That seems a little heavy-handed, a clear case of using a sledge hammer to kill a fly.

Another is to show that continuous maps between smooth manifolds do induce homo- morphisms on de Rham cohomology. This is of course true by de Rham's theorem, but can be proved much more easily, by approximation (at least on σ-compact manifolds), and using partitions of unity.

F irst show that on any σ-compact manifold there exist Riemannian metrics that are *controlled at infinity* in the sense that there exists $\rho_0 > 0$ such that any pair of points distance $< \rho_0$ apart are joined by a unique geodesic.

N ext show that if X and Y are σ-compact manifolds with Riemannian metrics, then every continuous $f : X \to Y$ can be uniformly approximated by C^∞-maps, and if the metric of Y is controlled at infinity, then any two approximations within $\rho_0/2$ are smoothly homotopic.

Page 16 In the last paragraph before Section 1.7, "Thus our map . . . " would be better as "Thus, by the reflection principle, our map . . . ." (The reflection principle says that if $U \subset \mathbb{H}$ is open and $f : U \to \mathbb{C}$ is an analytic function such that $\operatorname{Im} f(z) \to 0$ when $\operatorname{Im} z \to 0$, then $f(z) = \overline{f(\bar{z})}$ extends f analytically to $U \cup U^* \cup (\overline{U} \cap \mathbb{R})$. We do not have to assume that f extends continuously to $\overline{U} \cap \mathbb{R}$.)

Page 134 2 lines before Definition 4.5.1: The skew also becomes large when one vertex of a triangle tends to infinity, but we are only interested in small triangles.

Page 145 In Definition 4.5.13, W is not defined. It should be: " ... if every point of X has a neighborhood W such that for any three distinct points x, y, $z \in W$ . . . "

Page 203 2 lines after Proposition 5.2.12, $|x|$, not $\sup |x|$; 4 lines after Proposition 5.2.12, "or it has a maximum" should be "or $|g|$ has a maximum".

Page 209 Line 10: One reader thought $q(z)dz^2$ is a 2-form. It is not a 2-form, it is a quadratic differential.

Page 214 Perhaps I should have elaborated on the last sentence before Figure 5.3.7:

... for each critical point of q' in Y and each critical trajectory emanating from it, mark the first intersection of that trajectory with J (if it exists), as illustrated in Figure 5.3.7. (We will see right after equation 5.3.12 that it does exist.)

Page 215 Two paragraphs are repeated from page 214.

Page 226 A minus sign is missing in line 7: " ... defined over $(\mathbb{P}^1 \times \mathbb{P}^1)\Delta$" should be " ... defined over $(\mathbb{P}^1 \times \mathbb{P}^1) - \Delta$".

Page 293–294 There is some confusion here with Γ and G. In the first line of Section 6.12, Γ should be omitted, to avoid confusion with the Γ of Notation 6.12.1; i.e., replace "groups $\Gamma \subset \mathrm{Aut}(\mathbb{P}^1)$" by "subgroups of $\mathrm{Aut}(\mathbb{P}^1)$".

Page 306 The comment (after Exercise 7.3.2) that "I don't know how to continue the proof using this approach" reflected my ignorance. Curt McMullen points out that it is known that there is a constant $C(g)$ such that on every Riemann surface of genus g there is a maximal multicurve whose longest curve has length $\leq C(g)$. See Chapter 5 of Peter Buser's book *Geometry and Spectra of Compact Riemann Surfaces*, where C(g) is called "Bers' constant".

Page 342 Period missing at the end of the first sentence. Beginning of second paragraph: $\mathbb{R}/\mathbb{Z}$, not $\mathbb{R}/Z$

Page 343 Lemma A2.3: $f \circ \gamma_0 = \gamma_1$, not $f \circ \gamma_1 = \gamma_2$. Proof of Lemma A2.3, 4th line: "may intersect $\gamma_0(S^1)$", not "may intersect γ_0"

Page 352 Formula A3.11 is missing parentheses; it should be

$$0 \to H_0(\gamma_i) \to H_0(X_{i-1}) \oplus H_0(T_i) \to H_0(X_i) \underset{0}{\to}$$
$$\underset{0}{\to} \underset{\mathbb{Z}}{H_1(\gamma_i)} \to H_1(X_{i-1}) \oplus \underset{\mathbb{Z}^2}{H_1(T_i)} \to H_1(X_i) \to 0.$$

A parenthesis is missing from the last line of the proof, as well; "$\dim H_1(X_i) = \dim(H_1(X_{i-1}) + 1$" should be "$\dim H_1(X_i) = \dim(H_1(X_{i-1})) + 1$".

Page 354 Equation A4.5: The left side should be $\mu_J(i(a+ib) \otimes x)$, not $\mu_J(i(a+ib)\otimes$

Page 355 In equation A4.10, J is not the J of equation A4.3. The paragraph should be replaced by:

A better way to say this is to consider the open subset

$$\mathcal{K}(TM) \subset \mathrm{Gr}^{\mathbb{C}}(\mathbb{C} \otimes_{\mathbb{R}} TM)$$

of pairs (x, K_x) with $x \in M$ and K_x an n-dimensional $\mathbb{C}$-subspace of $\mathbb{C} \otimes_{\mathbb{R}} T_xM$ such that $K_x \cap \overline{K}_x = \{0\}$. Since the Grassmanians form a C^∞ bundle of complex manifolds, $\mathcal{K}(TM)$ is also a C^∞ bundle of complex manifolds over M. The space $S^k(M, \mathcal{K}(TM))$ of C^k-sections of the bundle $\mathcal{K}(TM)$ over M is the space of almost-complex structures on M of class C^k.

(Note that we have replaced "family" by "bundle" in two places.)

Page 357 Three lines before equation A4.16: $K \subset \mathbb{C} \otimes TM$ should be $K \subset \mathbb{C} \otimes_{\mathbb{R}} TM$.

Page 386 Line 3, missing word: "because *they* are by far ... ."

Page 392 Third line of Section A7.4: "more real than", not "more real that".

Page 392 End of third paragraph of Section A7.4: $H(X, \mathcal{O}_X^*)$, not $H^(X, \mathcal{O}_X^*)$

Page 397 The proof for proposition A7.5.6 is incorrect. Here is a corrected version: Proof Choose an exhaustion $\mathrm{V}_1 \subset V_2 \subset \cdots \subset U$ of U by open sets such that each V_i is relatively compact in V_{i+1}. Let U_i be the union of V_i and all the compact components of $U - V_i$; then every component of $\mathbb{C} - U_i$ contains points that are not in U. Further, choose C^∞ functions h_i on $\mathbb{C}$ that are identically 1 in U_i and identically 0 on $\mathbb{C} - U_{i+1}$.

Given $\alpha \in A^{0,1}(U)$, by equation A7.5.9 we can find $\beta_n \in A^{0,0}(U)$ such that $\bar{\partial}\beta_n = h_n$. Since we can write

$$\beta_n := \beta_0 + (\beta_1 - \beta_0) + \cdots + (\beta_n - \beta_{n-1}), \qquad \text{A7.5.13}$$

it is tempting to set $\beta = \beta_0 + \sum_{k=1}^{\infty}(\beta_{k+1} - \beta_k)$; unfortunately, the series does not converge. Instead, note that $\beta_{k+1} - \beta_k$ is analytic on a neighborhood of U_k, and every component of $\mathbb{C} - U_k$ contains points not in U. Thus by the Runge approximation theorem, there exist rational functions p_k analytic in U such that

$$\sup_{z \in \overline{U}_k} |\beta_{k+1}(z) - \beta_k(z) - p_k(z)| \leq \frac{1}{2^k} \qquad \text{A7.5.14}$$

Now the series

$$\beta := \beta_0 + (\beta_1 - \beta_0 - p_0) + \beta_2 - \beta_1 - p_1) + \cdots \qquad \text{A7.5.15}$$

converges uniformly on compact subsets of U, and β satisfies $\bar{\partial}\beta = \alpha$. $\square$

Page 412 Theorem A9.14: Here X is supposed to be a complex manifold of complex dimension n.

Page 419 Corollary A10.2.6: $\mathcal{O}(-\mathrm{div}(s))$ should be $\mathcal{O}(+\mathrm{div}(s))$.

Page 445 Two references were omitted:

[99] A. Weil, *Modules des surfaces de Riemann*, Bourbaki seminar **168** (1957–1958).

[101] S. Wolpert, *On the symplectic geometry of deformations of a hyperbolic surface*, Annals of Mathematics, **117**, no. 3 (1983), 207–234.

There is no reference [100].

Index entries There should be entries for