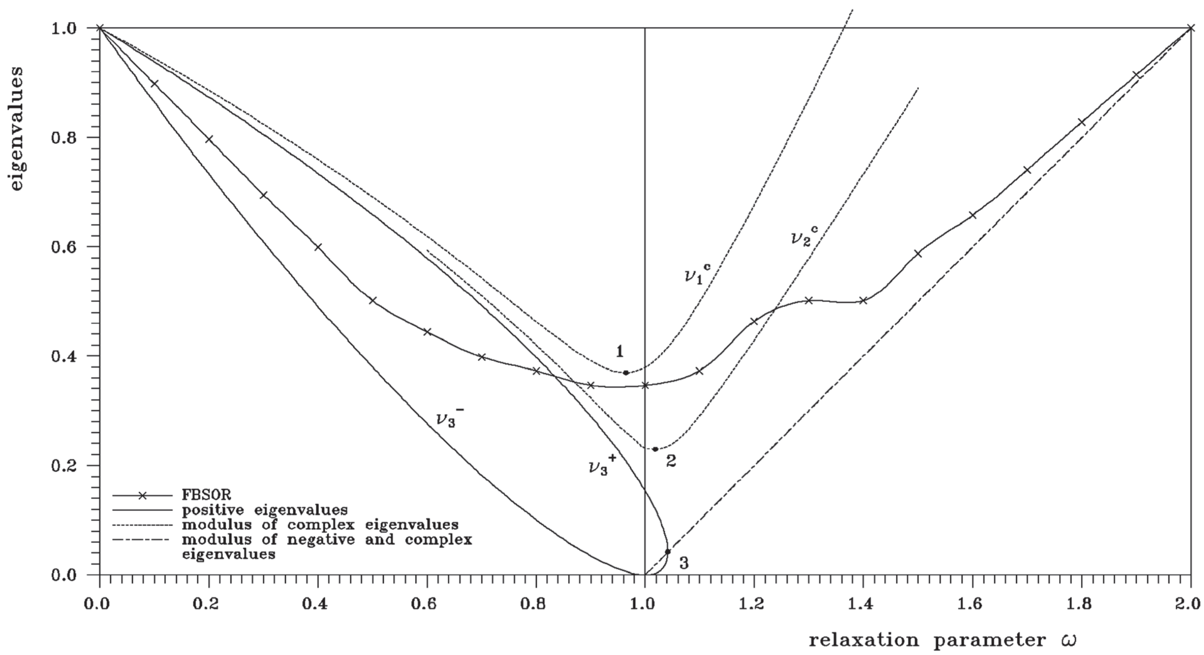

FIGURE 4.2.10. Dominant eigenvalues of the SOR iteration matrix $\mathcal{L}_\omega$ as a function of ω in example 4.2.24 (nonmonotone matrix $\mathbf{A}_8$)

EXAMPLE 4.2.25 Let $\mathbf{A}_9$ be the nonsymmetric matrix

$$\mathbf{A}_9 = \begin{bmatrix} 10 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ -2 & 10 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ -2 & -2 & 11 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ -2 & -2 & -2 & 12 & 1 & 1 & 1 & 1 & 1 & 1 \\ -2 & -2 & -2 & -2 & 13 & 1 & 1 & 1 & 1 & 1 \\ -2 & -2 & -2 & -2 & -2 & 14 & 1 & 1 & 1 & 1 \\ -2 & -2 & -2 & -2 & -2 & -2 & 15 & 1 & 1 & 1 \\ -2 & -2 & -2 & -2 & -2 & -2 & -2 & 16 & 1 & 1 \\ -2 & -2 & -2 & -2 & -2 & -2 & -2 & -2 & 17 & 1 \\ -2 & -2 & -2 & -2 & -2 & -2 & -2 & -2 & -2 & 18 \end{bmatrix}.$$

Since $\mathbf{A}_9$ is not consistently ordered, the two iteration matrices have different eigenvalue spectra: for $\mathcal{L}_1^f$ the dominant eigenvalue $\lambda_1 = -0.970649$ is negative and the remaining eigenvalues are complex with $\lambda_{10} = 0$ and $|\lambda_2| = 0.076897$, whereas the matrix $\mathcal{L}_1^b$ has mainly a complex spectrum such that $|\lambda_1| = 0.299836$, $|\lambda_2| = 0.124495$, $\lambda_9 = 0$, and $\lambda_{10} = -0.064164$. Thus in this example, the spectral radii of $\mathcal{L}_1^f$ and $\mathcal{L}_1^b$ differ significantly. The behavior of dominant eigenvalues of $\mathcal{L}_\omega^f$ and $\mathcal{L}_\omega^b$ as a function of ω is shown in figures 4.2.11 and 4.2.12.

The forward sweep is represented by Model M; the minimum of $\varrho(\mathcal{L}_\omega^f)$ occurs at the point 2 with $\varrho(\mathcal{L}_\omega^f) \approx 0.19$ for $\omega = 0.826$. As shown in figure 4.2.11, at the point 2 the dashed curve of the modulus of the negative eigenvalue ν_1^+ cuts the dotted curve of the modulus of the complex eigenvalue ν_2^c.