about 20 percent, whereas overestimating decreases it.

The coefficients E_{tot} are smaller than those given in table D.1.4 for Problem A(c), which means that in this problem RecOLA algorithms are more competitive with RecSLOR.

Algorithm	$\overline{\omega}$	It	Fl $\times 10^6$	Fl_{tot} $\times 10^6$	E_{tot}
	1.1822	1396	38.72	42.69	1.20
RecOLA-(0,0)	<u>1.1823</u>	1029	28.54	32.51	0.92
	1.1824	1059	29.37	33.34	0.94
	1.1228	825	18.11	29.42	0.83
RecOLA-(1,1)	<u>1.1229</u>	770	24.20	27.69	0.78
	1.1230	759	22.91	26.40	0.74
	1.0876	729	36.39	43.83	1.24
RecOLA-(2,2)	<u>1.0877</u>	585	29.20	36.64	1.03
	1.0878	533	26.61	34.05	0.96
	1.1498	933	18.11	20.27	**0.57**
RecOLA9R-(1,1)	<u>1.1499</u>	883	17.14	19.30	0.56
	1.1500	828	16.27	18.43	**0.52**
RecSLOR	1.9867	2318	34.29	35.48	1.00

TABLE D.1.17. Problem D ($43 \times 43 = 1849$ mesh points) , $\overline{\omega}_f = \overline{\omega}_b = \overline{\omega}$. The RecOLA9R-(1,1) algorithm provides the best results, and overestimating $\overline{\omega}$ is better than underestimating.

Among all the RecOLA algorithms considered, RecOLA9R-(1,1) with the double SOR, used to solve reduced systems, still provides the best results. In this problem the arithmetical effort is half that for RecSLOR. RecOLA algorithms with forward or backward SOR are inefficient in this problem, compared to the double SOR.

D.2 Non-self-adjoint problems

In this section, we analyze the convergence properties of semi-explicit prefactorization RecOLA algorithms when solving the non-self-adjoint Problems E, F, G, and H, defined in section 3.5, with nonzero right-hand sides for the majority of them. We consider the following algorithms: RecOLA-(0,0), RecOLA-(1,1), RecOLA-(2,2), and RecOLA9R-(1,1). Results obtained are compared to those obtained with RecSLOR.

The convergence analysis is based on investigating the relationship between the relative errors (4.212), (4.214), and (4.215), used usually as stopping tests, and the residual error $\mathbf{r}^{(t)} = \mathbf{c} - \mathbf{A}\boldsymbol{\phi}^{(t)}$ and the true error $\mathbf{e}^{(t)} = \boldsymbol{\phi}^{(t)} - \boldsymbol{\phi}$.

Since in problems considered in this section the right-hand sides are generated from the equation $\mathbf{A}\boldsymbol{\phi} = \mathbf{c}$ by assuming an exact solution $\boldsymbol{\phi}$, it is easy to determine the true error $\mathbf{e}^{(t)}$, which is the most reliable measure of accuracy for the obtained solution.

Problem E

Variant E(a) with $M = N = 31$ and $p_1 = p_2 = p_3 = 0$

Since equation (3.152) reduces to a self-adjoint problem, i.e., a boundary-value problem representing the Poisson equation, the associated matrix $\mathcal{B}_1 = \mathbf{K}^{-1}(\mathbf{L}+\mathbf{U})$ is nonegative and consistently ordered. For RecSLOR, ω_{opt} can be determined using the Sigma-SOR algorithm, as shown in table A.2.1, and for prefactorization algorithms, it can be determined using the OMEST procedure.

Table D.2.1 gives results of spectral radius computations to six significant digits, obtained using the starting vector $\boldsymbol{\phi}^{(0)} \equiv [0,0]$.

Algorithm	ω_f	ω_b	ϱ_ω	It_p	Fl_p $\times 10^6$	$\overline{\omega}$	$\bar{\varrho}_\omega$
	1.	1.	0.940758	60	0.634		
	1.10	1.10	0.851229	53			
RecOLA-(0,0)	1.11	1.11	0.825056	45	1.845	1.1326	0.6098
	1.12	1.12	0.784987	30			
	1.	1.	0.882744	35	0.437		
	1.05	1.05	0.793978	39			
RecOLA-(1,1)	1.06	1.06	0.756332	33	1.585	1.0804	0.4926
	1.07	1.07	0.697095	25			
	1.	1.	0.796179	24	0.530		
	1.035	1.035	0.658212	25			
RecOLA-(2,2)	1.04	1.04	0.617394	22	1.790	1.0504	0.3822
	1.045	1.045	0.558831	22			
	1.	1.	0.813113	28	0.229		
	1.06	1.06	0.676684	29			
RecOLA9R-(1,1)	1.07	1.07	0.628024	20	0.757	1.0865	0.3980
	1.08	1.08	0.550606	26			
	–	1.	0.980923	269	1.551		
	–	1.5	0.941560	117			
RecSLOR	–	1.6	0.919754	80	0.687	1.7573	0.7573
	–	1.7	0.874632	71			

TABLE D.2.1. Problem E(a) ($31 \times 31 = 961$ mesh points), $\overline{\omega}_f = \overline{\omega}_b = \overline{\omega}$

Table D.2.2 gives error norms for solutions. The results obtained for RecSLOR are quoted from tables A.2.1 and A.2.2. The computational work $FL_{p(tot)}$ is the sum of the values of Fl_p obtained for three chosen ω for RecOLA algorithms and the average of the values of Fl_p obtained for three chosen ω in the case of RecSLOR.

As is seen in table D.2.2, using values of ω smaller and larger than its optimum does not significantly increase the computational work Fl compared to using $\overline{\omega} \equiv \overline{\omega}_f = \overline{\omega}_b$. Underestimating the optimum value of ω increases Fl more than overestimating it by the same amount.

The convergence behavior of these semi-explicit prefactorization algorithms is similar to that observed for the explicit prefactorization algorithms discussed in chapter 5. For problems in which $\varrho_{\omega=1} \leq 0.90 \div 0.95$, which is satisfied in this case, the rate of convergence is relatively insensitive to accurate values of $\overline{\omega}_f$ and $\overline{\omega}_b$. The following examples will show that obtaining the solution using a rough estimate of $\overline{\omega}_f$ and $\overline{\omega}_b$ or frequently with $\omega_f = \omega_b = 1$, may be much less time-consuming than using accurate values of $\overline{\omega}_f$ and $\overline{\omega}_b$. In the case of RecSLOR, $\overline{\omega}$ can be estimated with the Sigma-SOR algorithm, using the value of the spectral radius computed to three or four significant digits after the decimal point.