The convergence behavior of the R9MSLOR-1L and R9MSLOR-1P algorithms, used to solve a higher-order finite-difference approximation represented by the 9-point formula (6.58) and depicted in figure C.1.4, bottom, is similar to that observed in the previous algorithms based on 5-point finite-difference approximation. Only the point-relaxed model R9MSLOR-1P is computationally useful.

Variant A(b) with $M = N = 30$

Tables C.1.2 and C.1.3 summarize results for modified line algorithms with point relaxation; results are also given for RecSLOR and RecSLOR9. Table C.1.2 concerns estimating the optimum values of ω_{opt} for each algorithm. For RecSLOR and RecSLOR9, where the Sigma-SOR algorithm is used to determine ω_{opt}, the quantity $Fl_{p(tot)}$ is the average of three computations of ϱ_ω.

Algorithm	ω	ϱ_ω	It_p	Fl_p $\times 10^6$	$Fl_{p(tot)}$ $\times 10^6$	ω_{opt}
RMSLOR-1P	1.26	0.875324	51	0.367		
	1.28	0.851766	43	0.310	1.116	1.3198
	1.3	0.814105	61	0.439		
	1.	0.962561	125	0.670		
RMSLOR-2aP	1.22	0.866726	54	0.486		
	1.24	0.826866	39	0.351	1.332	1.2849
	1.26	0.777905	55	0.495		
	1.	0.953516	116	0.835		
RMSLOR-3P	1.18	0.845626	45	0.486		
	1.2	0.814607	32	0.346	1.307	1.2346
	1.22	0.763065	44	0.475		
	1.	0.940840	107	0.963		
RMSLOR-4P	1.16	0.821025	44	0.515		
	1.18	0.781333	35	0.410	1.498	1.2089
	1.2	0.706931	49	0.573		
	1.	0.928353	91	0.901		
RecSLOR	1.6	0.914217	88	0.634		
	1.65	0.895273	57	0.410	0.528	1.7505
	1.7	0.864476	75	0.540		
	1.	0.979686	255	1.377		
R9MSLOR-1P	1.32	0.880365	60	0.540		
	1.34	0.860613	49	0.441	1.494	1.3865
	1.36	0.831222	57	0.513		
	1.	0.967395	222	1.598		
RecSLOR9	1.6	0.914140	77	0.832		
	1.65	0.895174	53	0.572	0.745	1.7504
	1.7	0.864332	77	0.832		
	1.	0.946787	110	0.990		

TABLE C.1.2. Problem A(b) ($30 \times 30 = 900$ mesh points)

For the modified line algorithms, the OMEST procedure described in subsection 4.2.4 is used: for three values ω_1, ω_2, and ω_3, chosen in such a way that

$$1 < \omega_1 < \omega_2 < \omega_3 < \omega_{opt},$$

the value of ϱ_ω is computed using the power method, to six significant digits after the decimal point; It_p is the corresponding number of power method iterations, and Fl_p is the computational work. For these algorithms, the values of $Fl_{p(tot)}$ are the sum of the computational work for three computations of ϱ_ω.

Table C.1.3 shows results obtained for ϱ_ω, using the starting vector $\boldsymbol{\phi}^{(0)} \equiv [1,1]$ and the stopping test $\|\boldsymbol{\phi}^{(t)}\|_\infty \leq 10^{-12}$, where It is the number of iterations, Fl is the corresponding computational work, and $Fl_{tot} = Fl + Fl_{p(tot)}$.

Algorithm	ω	ϱ_ω	It	Fl $\times 10^6$	Fl_{tot} $\times 10^6$	R_∞	E_∞	E_t	E_{tot}
RMSLOR-1P	1.3198	0.6843	86	0.619	1.735	0.3795	1.32	1.32	1.28
RMSLOR-2aP	1.2849	0.6506	77	0.693	2.025	0.4299	1.50	1.20	1.50
RMSLOR-3P	1.2364	0.6165	69	0.745	2.052	0.4837	1.69	1.12	1.52
RMSLOR-4P	1.2089	0.5848	62	0.725	2.223	0.5365	1.87	1.15	1.64
RecSLOR	1.7505	0.7505	117	0.824	1.352	0.2870	1.	1.	1.
R9MSLOR-1P	1.3865	0.6994	94	0.846	2.342	0.3653	1.25	1.5	1.17
RecSLOR9	1.7504	0.7504	117	1.264	2.009	0.2871	1.	1.	1.

TABLE C.1.3. Problem A(b) ($30 \times 30 = 900$ mesh points). For MSLOR algorithms with point modification, the total coefficient of efficiency E_{tot} is the ratio of Fl_{tot} for a given algorithm to Fl_{tot} for RecSLOR. For R9MSLOR-1P, it is the ratio of Fl_{tot} for R9MSLOR-1P to that of RecSLOR9.

Variant A(c) with $M = N = 60$

The results of computations, obtained in the same way as those in variant A(b), are summarized in tables C.1.4 and C.1.5.

In variants A(b) and A(c), the values of E_{tot} for modified algorithms are greater than 1 because much of the computational work is consumed by determining ω_{opt} to four significant digits after the decimal point.

As seen in table C.1.3, the values of $\varrho_{\omega=1}$ are equal to about 0.95. We saw in section 5.3 that for such problems, the rate of convergence is insensitive to the accurate estimate of ω_{opt}: rough estimates of ω_{opt}, to two significant digits after the decimal point, are sufficient in practice, which allows us to significantly reduce the computational work while decreasing the values of E_{tot} below 1.

To illustrate this effect, consider in table C.1.5 the RMSLOR-4P solution obtained for $\omega_{opt} = 1.2505$ with $It = 124$ and $Fl = 5.8 \times 10^6$ flops. Using $\omega = 1.24$, the solution requires $It = 201$ and $Fl = 9.4 \times 10^6$ flops, but for $\omega = 1.26$ it requires $It = 134$ and $Fl = 6.3 \times 10^6$ flops. Thus the increased values of Fl are still much less than Fl_{tot} given in table C.1.5, and overestimating ω_{opt} is better than underestimating by the same amount. However, when using RecSLOR, it is necessary to make a more accurate estimate of ω_{opt}.