Problem D

Tables A.1.26 and A.1.27 summarize the results of computations using RecSOR, RecSLOR, and RecSOR9R for determining ω_{opt} in Problem D. In Problem C(a), the coefficients D differ by two orders in magnitude (see figure 3.5.2), which implies that values of the subdominance ratio σ are very close to 1 in the algorithms considered. The discontinuity of coefficients in Problem D is much weaker than that in Problem C; therefore, determining ω_{opt} by means of the Sigma-SOR algorithm is much cheaper compared to the total computational work; as is seen in table A.1.27, the values of $Fl_{p(avr)}$ are about 20 times less than those for Fl, obtained by computing $\varrho_{\omega=1}$.

Algorithm	ω	ϱ_ω	It_p	Fl_p $\times 10^6$	$Fl_{p(avr)}$ $\times 10^6$	ω_{opt}
RecSOR	1.90	0.999574	131	1.696		
	1.92	0.999461	128	1.657	1.890	1.99058
	1.94	0.999272	179	2.317		
	1.	0.999978	647	5.982		
RecSLOR	1.88	0.999296	74	1.095		
	1.89	0.999228	84	1.243	1.194	1.98671
	1.9	0.999146	84	1.243		
	1.	0.999955	692	7.677		
RecSOR9R	1.88	0.999050	69	0.702		1.98457
	1.89	0.998958	79	0.803	0.753	1.98458
	1.9	0.998847	74	0.753		1.98459
	1.	0.999940	389	3.237		1.98463

TABLE A.1.26. Problem D ($43 \times 43 = 1849$ mesh points)

Algorithm	ω	ϱ_ω	It	Fl $\times 10^6$	$Fl_{p(avr)}$ $\times 10^6$	R_∞	E_∞	E_t	E_{obs}
RecSOR	1.99058	0.99058	1781	23.05	1.890	0.00946	0.707	0.808	0.804
RecSLOR	1.98671	0.98671	1253	18.53	1.194	0.01338	1.	1.	1.
RecSOR9R	1.98457		1308	13.30					1.393
	1.98458		1307	13.29	0.753				1.394
	1.98459		1305	13.27					1.397

TABLE A.1.27. Problem D ($43 \times 43 = 1849$ mesh points)

Table A.1.26 shows that for RecSOR9R, the assumed values of $\omega = 1.88,\ 1.89$ and 1.9 for Sigma-SOR algorithm computations provide nearly the same values of ω_{opt}. This means that for this problem, the deviation of behavior of the spectral radius as a function of ω from the consistently ordered case is insignificant. The value of E_{obs} shown in table A.1.27 for RecSOR9R is less than in the former problems.

The number of iterations as a function of ω shown in figure A.1.15 for RecSOR, RecSLOR, and RecFBSLOR, using the starting vector $\boldsymbol{\phi}^{(0)} \equiv [-0.5, 0.5]$, is quite different from that shown in figure A.1.14 for Problem C(b). An unusual behavior is observed for RecFBSLOR, where the minimal number of iterations is achieved with underrelaxation, with $\omega = 0.13$. The RecSOR9R, RecFBSOR, and RecFBSLORP algorithms are inefficient for this problem, with $\boldsymbol{\phi}^{(0)} \equiv [-0.5, 0.5]$.

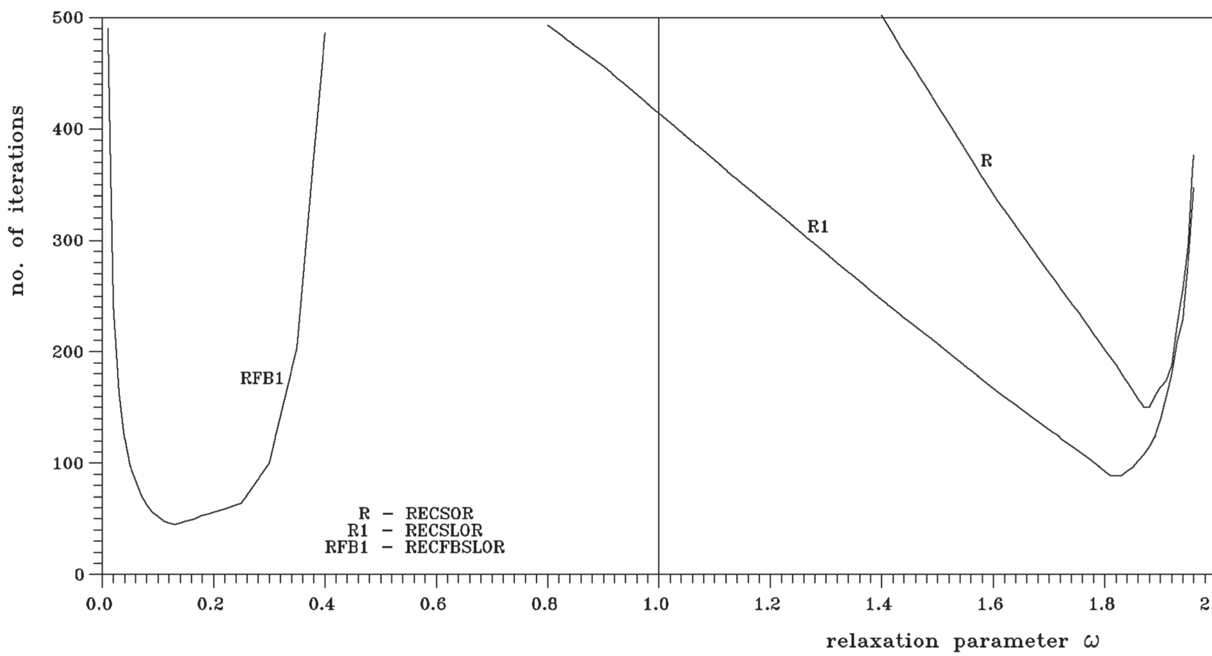

FIGURE A.1.15. The of number of iterations as a function of ω in the RecSOR, RecSLOR, and RecFBSLOR algorithms for Problem D, using the starting vector $\boldsymbol{\phi}^{(0)} \equiv [-0.5, 0.5]$.

The results obtained for starting vectors $\boldsymbol{\phi}^{(0)} \equiv [1, 1]$ and $\boldsymbol{\phi}^{(0)} \equiv [-0.5, 0.5]$ are compared in table A.1.28. For RecSOR and RecSLOR, using $[-0.5, 0.5]$ instead of $[1, 1]$ reduces the computational work by a factor of more than ten.

Algorithm	$\boldsymbol{\phi}^{(0)} \equiv [1,1]$				$\boldsymbol{\phi}^{(0)} \equiv [-0.5,0.5]$			
	ω	It	Fl $\times 10^6$	E_{obs}	ω	It	Fl $\times 10^6$	E_{obs}
RecSOR	1.99058	1781	23.05	0.804	1.87	150	1.941	0.678
RecSLOR	1.98671	1253	18.53	1.	1.82	89	1.316	1.
RecFBSLOR					0.13	45	1.331	0.989

TABLE A.1.28. Problem D ($43 \times 43 = 1849$ mesh points)

The rate of convergence of RecFBSLOR is not acceptable using $\boldsymbol{\phi}^{(0)} \equiv [1, 1]$: the