Teichmüller Theory and Applications to Geometry, Topology, and Dynamics

Volume 2: Surface Homeomorphisms and Rational Functions

Errata and Notes, complete as of February 27, 2020

We thank Alastair Fletcher, Wolf Jung, Dan Margalit, and Ahmad Rafiqi for their contributions to this list.

Mathematical errata

Page 43 Next-to-last line of caption: $.\overline{9} = 1.\overline{0}$, not $.9 = 1.\overline{0}$

Page 22 End of first paragraph of subsection on stretch factors: "in particular the eigenvalues of symmetric matrices with integer or rational coefficients are totally real", not "in particular the eigenvalues of symmetric real matrices (or complex Hermitian matrices) are totally real"

Page 32 [added March 29, 2017] Two lines after Definition 8.6.1: "a positive integer matrix", not "a positive matrix".

Page 34 [added March 29, 2017] Four lines before the remark: "The 1-form $\operatorname{Re}\omega_q$", not "The 1-form ω_q".

Page 38 Proof of Proposition 9.1.4, part 2, second line: "the sequence $n \mapsto p^{\circ n}$ of iterates on the complement of X", not "the sequence $n \mapsto p^{\circ n}$ of iterates".

Page 63 Theorem 9.4.1 could be expanded by changing "a finite orbit containing a repelling cycle" in part 2 to "a finite orbit containing a repelling or parabolic cycle". The proof would have to be adapted.

Page 85 Second paragraph: "the norm $\|D\sigma_f(\tau)\|$ of the derivative can be bounded in terms of the projection ... ", not "the norm $\|D\sigma_f(\tau)\|$ of the derivative depends only on the projection ... ".

Page 92 Remark: "unimodal", not "unimodular", and footnote no. 2 should be deleted.

Page 93 The claim that part 1 of Proposition 10.4.3 0 is a special case of Theorem 9.3.2 is not correct, since that theorem applies only to the interior. It's not hard to fix the proof. Either we invoke Theorem 9.4.1 to say that J_p is locally connected, or we adapt the proof of Theorem 9.4.1 to the case of hyperbolic polynomials.

Page 110 Caption to Figure 10.4.9: $\theta = \overline{.001100110100} = \dfrac{820}{4095}$, not $\theta = \overline{.0010011010010} = \dfrac{594}{4095}$.

Page 113 Corollary 10.4.17 is not stated correctly. The corrected algorithm is written in Bruin, Kaffl, and Schleicher (*Symbolic Dynamics of Quadratic Polynomials*, monograph in preparation), who attribute it to Mary Rees. We are replacing everything from "Note that since θ has period k ... "(second sentence of the paragraph immediately before the corollary) to the end of the section by the following:

Let θ' be the companion of θ; set $\theta_i := 2^{i-1}\theta$ and $\theta'_i := 2^{i-1}\theta'$. Angles will be viewed as elements of $\mathbb{R}/\mathbb{Z}$; "preimage" means preimage under $t \mapsto 2t$, i.e., angle doubling; every interval in $\mathbb{R}/\mathbb{Z}$ has two disjoint preimages.

Define intervals $I_k, I_{k-1}, \dots, I_1$, as follows: the interval I_k is the preimage of $[\theta/2, (\theta+1)/2]$ that does not contain θ_k; it is bounded by two angles φ'_k and φ''_k, with $\varphi'_k = \theta_k$ periodic and φ''_k preperiodic. Then for all $j = 1, \dots, k-1$ the interval I_j is the preimage of I_{j+1} bounded by preimages φ'_j of φ'_{j+1} and φ''_j of φ''_{j+1}, chosen so that φ''_j is on the side of the dividing line specified by $\Sigma^j_\theta(\theta)$.

Corollary 10.4.17 *The angle θ' is the unique angle of period k in the interval I_1.*

PROOF Observe that $\theta'_k \in I_k$. Further φ''_j always lands on a point of the boundary of the component U_j of $\operatorname{int} K_\theta$ containing $x_j = p^{\circ j}_\theta(0)$. It then follows that $\theta'_j \in I_j$, and therefore $\theta' = \theta'_1 \in I_1$. It is the unique periodic angle in the interval because $|I_1| = 1/2^k$ and the angles periodic of period k are spaced $1/(2^k - 1)$ apart. □

Example 10.4.18 Let us find the companion of $\theta = 4/15 = .\overline{0100}$. Make a table

θ_1	$.\overline{0100}$	B	$I_1 = [.00110\overline{0100}, .00111\overline{0100}]$
θ_2	$.\overline{1000}$	B	$I_2 = [.0110\overline{0100}, .0111\overline{0100}]$
θ_3	$.\overline{0001}$	A	$I_3 = [.110\overline{0100}, .111\overline{0100}]$
θ_4	$.\overline{0010}$	?	$I_4 = [.10\overline{0100}, .11\overline{0100}]$

We see that indeed $.\overline{0011} = 3/15$ is the only angle that is periodic of period 4 and is contained in I_1. △

Page 118 [new Feb. 27, 2020] Equation 10.5.8: on the right side, the p^{o2} in the denominator should be $p^{o(k-1)}$.

Page 119 Figure 10.5.3, lower right: the line marked θ should not extend into the pink (nor should the lines marked $\theta/2$ and $(\theta+1)/2$, but it's not clear they do).

Page 175 Part 3 of the remark should be: If we replace "homeomorphism" by "diffeomorphism", the result remains true for S^2 and S^3, but it is false in higher dimensions. For S^3 the result is very hard [18].

Page 227 Last paragraph: replace "if a Thurston map f is unicritical, or if it is a topological polynomial such that $\mathrm{Crit}_f \subset \mathrm{P}_f$" by "if a topological polynomial f is unicritical or satisfies $\mathrm{Crit}_f \subset \mathrm{P}_f$".

Notes and clarifications

Page 92 Definition 10.3.2: The kneading sequence defined here is what Dierk Schleicher calls an "itinerary"; he uses "kneading sequence" only for the itinerary of the critical value or of the corresponding external angle.

Page 93 Theorem 10.3.3: In the case where θ is in $\mathbb{Q}^{\mathrm{even}}/\mathbb{Z}$ and there is a Thurston obstruction, the points X_i do converge under iteration of σ_f, but they do not remain distinct. This was proved in greater generality by Nikita Selinger.

Page 114 Proposition and Definition 10.5.4: "contained in the closed disc", not "contained in the disc".

Page 201 [added April 21, 2019] In equation C5.12, e^t should be $e^{t/2}$ and e^{-t} should be $e^{-t/2}$.

Non-mathematical errata

Page x Next-to-last line: "very time" should be "every time".

Page 22 We misspelled Yoccoz's first name. It is Jean-Christophe.

Page 102 Last line of caption to Figure 10.4.3: "three red external rays", not "three external red rays".

Page 148 Proof of Proposition 110.6.8: In the first line, "This is exactly", not "This exactly".

Page 154 Line immediately after equation 10.7.14: "$f^{-1}(Z)$ has $d|Z|$ elements", not "$f^{-1}(Z)$ has $d|Z|$ elements in its inverse".

Page 174 Second paragraph of the remark, last line: "the scope of", not "the the scope of".

Page 201 [added April 21, 2019] First line after equation C5.12: "preserves the subset", not "preserves he subset".

Page 249 First line of subsection *Similarities in the proofs*: "Many proofs in complex dynamics", not "Many proof in complex dynamics".

Page 251 Entry [18]: "Lecture Notes", not "Lectures Notes"