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Bold page numbers indicate a page where a term is defined, either formally or informally. *Page numbers in italics* indicate that the term is used in a theorem, proposition, lemma, or corollary.

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