B1
Glossary

Entries in this glossary are some terms used but not defined in the book. Text in brackets describes where the term is first used. The choice of what to include is somewhat arbitrary; I have particularly tried to include words whose usage in the mathematical literature is ambiguous (for instance, charts and local coordinates for manifolds). I have also tried to include words when several words in the literature describe the same concept: the reader may have studied the concept under a different name: cylinders and annuli for instance. I have also included notions that I feel are important, but don't seem to be in the curriculum in many places, such as "proper map" and "group action." Differential forms are a more delicate matter: I give some hints but nowhere near enough to bring a reader who doesn't know the topic up to speed.

Other entries are included because readers of early drafts of the book were puzzled by some bit of notation (such as why a cokernel is called a cokernel).

act freely, act transitively [Section 1.8] See group action.

annulus [proof of Lemma 1.4.3] Synonymous with cylinder; discussed in Section 3.2.

blow-up [Example 1.3.4] *Blowing up* a submanifold of a manifold is a construction from algebraic and analytic geometry, in which the submanifold is replaced by the projective space bundle of its normal bundle. To blow up a point x on an n-dimensional manifold X, choose an isomorphism φ of a neighborhood U of x to a neighborhood V of 0 in T_xX (or $\mathbb{R}^n$ if we are dealing with differentiable manifolds). For the notation $\mathbb{P}(\cdot)$, see the glossary entry on projective space. Let $V' \subset V \times \mathbb{P}(T_xX)$ be the subset

$$V' := \{\, (y \in V, L \in \mathbb{P}(T_xX) \mid y \in L \,\}$$

and define $\pi : V' \to V$ by $\pi(y, L) := y$. Then $\pi^{-1}(x) = \mathbb{P}(T_xX)$ and the map $\pi : V' - \pi^{-1}(x) \to V - \{x\}$ is an isomorphism. The blow-up $\widetilde{X}_x$ is the quotient of $X - \{x\} \sqcup V'$ by the equivalence relation that identifies $(y, L) \in V' - \pi^{-1}(x)$ with $\varphi^{-1}(y) \in X - \{x\}$. From this description it is easy to see that $\widetilde{X}_x$ is a smooth manifold with the desired properties.

braid [Section 5.2] Consider the space X_n of distinct n-tuples of points in $\mathbb{C}$. The *braid group* is the fundamental group of X_n. A *braid* with n strands is a closed path in X_n.

branch point [Example 6.3.6] If a map of surfaces is a local homeomorphism except at isolated points, these points are called *branch points*. The typical example is $z \mapsto z^2$, which has a branch point at 0. Synonymous with ramification point.

bundle [bundle map discussed in Section 4.8; tangent bundle discussed in Section 4.9] We use *bundle* as synonymous with "locally trivial bundle." A map $p: Y \to X$ is a locally trivial bundle if every $x \in X$ has a neighborhood U such that there exists an isomorphism $\varphi_U : p^{-1}(U) \to U \times p^{-1}(x)$ such that the diagram

$$\begin{array}{ccccc} p^{-1}(U) & & \xrightarrow{\varphi_U} & & U \times p^{-1}(x) \\ & p \searrow & & \swarrow pr_1 & \\ & & U & & \end{array}$$

commutes. A *trivialization* of p is a homeomorphism $h: Y \to X \times p^{-1}(x)$ for some x, such that

$$\begin{array}{ccccc} Y & & \xrightarrow{h} & & X \times p^{-1}(x) \\ & p \searrow & & \swarrow pr_1 & \\ & & X & & \end{array}$$

commutes. In most instances of interest, the fibers have extra structure, and the isomorphisms are required to preserve this structure; we then speak of a "bundle of ... .". A particularly important example is that of vector bundles, where the fibers are vector spaces. A trivialization of a "bundle of ... " is a trivialization that preserves whatever structure is given by "...".

bundle map [discussion following equation 4.8.17] If $p_1 : X_1 \to T$ and $p_2 : X_2 \to T$ are two bundles, then a map $f : X_1 \to X_2$ is a *bundle map* if $p_1 = p_2 \circ f$. If the X_i are bundles of something (vector spaces, Lie groups, complex manifolds, etc.), then f is required to preserve the relevant structure.

cardioid [introduction to Theorem 4.9.15] A *cardioid* is the plane curve obtained by marking a point on a circle, and rotating the circle on another circle of equal radius.

chart [Definition 1.2.1] See manifold.

closed form [proof of Proposition 1.6.1] See differential form.

cochain complex [Proposition A6.2.1] A *cochain complex* of Abelian groups is a sequence of Abelian groups $A^0, A_1, \dots,$ together with homomorphisms $d^i : A^i \to A^{i+1}$ such that $d^{i+1} \circ d^i = 0$ for every $i = 0, 1, 2, \dots$. The whole structure is often denoted $(A^\bullet, d^\bullet)$. The cohomology of the complex is

$$H^k(A^\bullet, d^\bullet) := \frac{\ker d^i : A^i \to A^{i+1}}{\operatorname{im} d^{i-1} : A^{i-1} \to A^i}.$$

In practice, the Abelian groups A^i often have more structure: they may be modules over some ring, or vector spaces over some field; in these cases the cohomology groups have the same structure.

codimension [proof of Proposition 7.4.15] A k-dimensional submanifold of a manifold of dimension n has *codimension* $n-k$. The same applies to a k-dimensional subspace of an n-dimensional vector space.

codomain [proof of Proposition 3.3.4] A map $f: X \to Y$ has domain X and *codomain* Y. The subset $f(X) \subset Y$ is called the *image* of f. The word "range" is ambiguous; some authors use it as synonymous with image, others as synonymous with codomain, and many use it for both.

cofinal [Example A7.2.5] In a partially ordered set $(X, \prec)$, a subset Z is *cofinal* if for every $x \in X$, there exists $z \in Z$ such that $x \prec z$. When taking direct and inverse limits, it is enough to consider a cofinal set of indices.

cohomology [Section 1.1] Included in the prerequisites, *cohomology* is a major topic in algebraic and differential topology, coming in many flavors. For the definition of De Rham cohomology, see the entry on differential forms. For cohomology of sheaves, see Appendix A7. Singular cohomology is covered in all textbooks on algebraic topology, for instance [56].

cokernel [Theorem 5.2.9] If $L: X \to Y$ is a linear transformation, then $\operatorname{coker} L = Y/L(X)$. Why the word "cokernel?" The answer comes from category theory.

The kernel of a morphism $f: A \to B$ is an object C with a morphism $g: C \to A$ such that $f \circ g = 0$ and whenever a morphism $h: D \to A$ satisfies $f \circ h = 0$, there exists a unique morphism $\alpha: D \to C$ such that $h = g \circ \alpha$.

The cokernel of a morphism $f': B' \to A'$ is an object C' together with a morphism $g': A' \to C'$ such that $g' \circ f' = 0$, and whenever a morphism $h': A' \to D'$ satisfies $h' \circ f' = 0$, there exists a unique morphism $\alpha': C' \to D'$ such that $h' = \alpha' \circ g'$. In the two corresponding diagrams

$$\begin{array}{ccccc} C & \xrightarrow{g} & A & \xrightarrow{f} & B \\ \alpha \uparrow & h \nearrow & & & \\ D & & & & \end{array} \quad \text{and} \quad \begin{array}{ccccc} C' & \xleftarrow{g'} & A' & \xleftarrow{f'} & B', \\ \alpha' \downarrow & h' \swarrow & & & \\ D' & & & & \end{array}$$

the second is exactly the first with all the arrows turned backwards.

complex dilatation [proof of Proposition 4.9.9] Let $U \subset \mathbb{C}$ be open. The *complex dilatation* of a map $f: U \to \mathbb{C}$ is

$$\frac{\partial f}{\partial \bar{z}} \Big/ \frac{\partial f}{\partial z}\,.$$