

FIGURE 10.5.4 The Mandelbrot set, shown in black. The external rays of M (also shown in black) land as described in Theorem 10.5.17. If (θ, θ') are companion angles, the external rays $R_M(\theta)$ and $R_M(\theta')$ meet at a point on the boundary of the component U_θ of the interior of M containing c_θ . This point $\hat{c}_\theta = \hat{c}_{\theta'}$ is called the *root* of U_θ . For instance, $R_M(1/3)$ and $R_M(2/3)$ meet at the point $-3/4$; this point is the root of the component $U_{1/3}$, which is a disc of radius $1/4$ centered at $c_{1/3} = -1$; this center corresponds to the basilica, seen already in Figure 10.4.4. External rays corresponding to different companion pairs land at distinct points. We have drawn all rays with angles in $\mathbb{Q}^{odd}/\mathbb{Z}$ of period ≤ 5 under angle doubling. In addition to the basilica, we have labeled the component whose center is the airplane $c_{3/7} = c_{4/7}$ and the component whose center is the rabbit $c_{1/7} = c_{2/7}$. Figure 10.5.5 on the opposite page is a cartoon emphasizing the combinatorics. The *main cardioid* above corresponds to the light blue region of the cartoon: the set of $c \in M$ such that p_c has an attracting fixed point; its root is at the cusp $1/4$. The disc centered at -1 corresponds to the green disc in Figure 10.5.5: the set of $c \in M$ such that p_c has an attracting cycle of period 2.

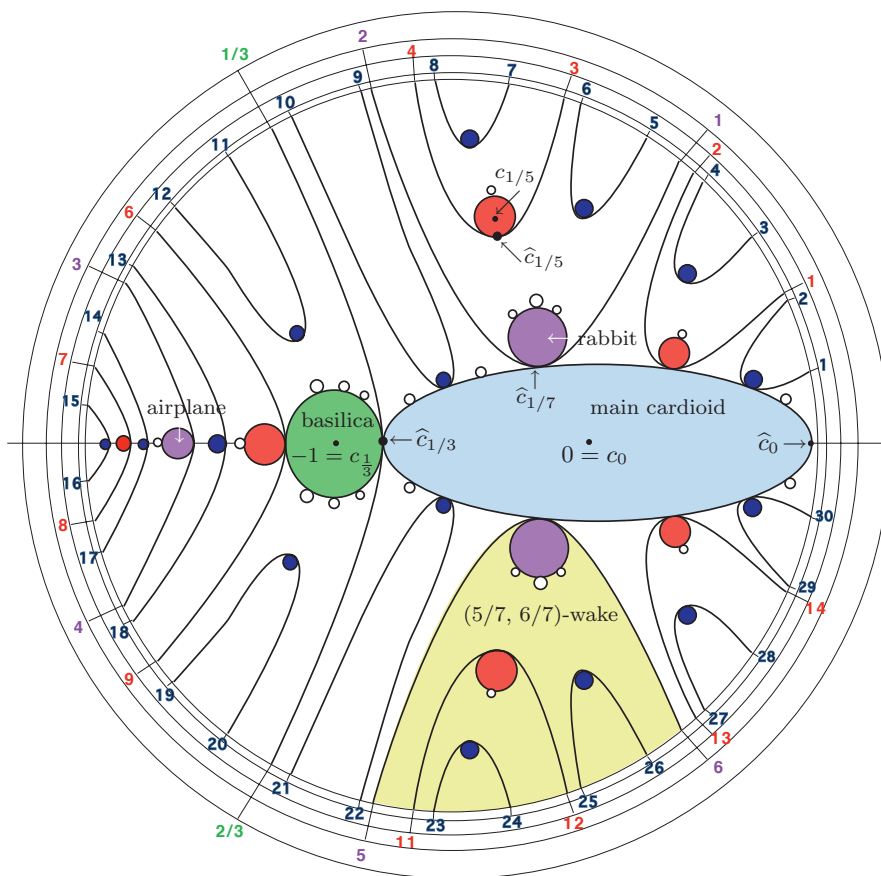


FIGURE 10.5.5 A cartoon of Figure 10.5.4, showing all rays with rational angles with denominators 3, 7, 15, and 31, i.e., all angles that repeat with period ≤ 5 under angle doubling. Light blue corresponds to period 1, green to period 2, purple to period 3, red to period 4, and dark blue to period 5. The colors also correspond to polynomials with an attracting cycle of the same period. We have marked c_θ and \hat{c}_θ for $\theta = 0$, $\theta = 1/3$, and $\theta = 3/15$; note from Figure 10.5.4 that $c_{1/3} = -1$ and $\hat{c}_{1/3} = -3/4$. The \hat{c} correspond to polynomials with parabolic cycles; $\hat{c}_{1/5}$ is a cusp. The yellow region is the $(5/7, 6/7)$ -wake; see Theorem 10.5.17. The components of the interior directly attached to the main cardioid (light blue) are satellites of the cardioid; those shown unattached are primitive. Both satellites and primitive components have further satellites, some of which are shown. The parts of M contained in a wake attached to the main cardioid form a *limb* of M ; the Mandelbrot set consists of the cardioid and its limbs. The yellow wake contains the $2/3$ -limb, since that wake is attached to the cardioid at internal angle $2/3$; see Definition 10.4.4. The four discs shown in that wake (there are infinitely many) are components of the interior of M , connected by filaments.