Teichmüller Theory and Applications to Geometry, Topology, and Dynamics

Volume 1 Teichmüller Theory

Infrequently Asked Questions

Q Do you ever construct the Teichmüller space of a torus in your book?

A I don't think it exists. There is a Teichmüller space of the once-punctured torus, but I don't believe in Teichmüller spaces of non-hyperbolic surfaces. Saying that there exists a Teichmüller space for a torus, or for a sphere with at most two punctures, introduces complications. I have deliberately treated only the case of Teichmüller spaces for hyperbolic surfaces to avoid making special constructions. For instance, it is much easier to understand $\mathrm{PSL}_2\,\mathbb{Z}$ as the mapping class group of the punctured torus than of the torus. I know of no case where there is information to be gotten from the torus that is not available from the Teichmüller space of a punctured torus.

Q The proof of proposition 5.3.4 starts with: "...cut X along all compact critical horizontal trajectories". Why are you allowed to assume that compact critical horizontal trajectories exist?

A I don't. If there aren't any, you don't cut.

Q page 208, line 8 – why is it true that a 1-form vanishes $2g-2$ times on a surface of genus 2?

A This is the Hopf index theorem, one of the prerequisites.

Q On page 214, in the first full paragraph, you write "mark the first intersection of that trajectory . . . ." The argument here seems to be assuming that the horizontal flow is recurrent, but you have not mentioned this.

A I am not sure that this is a mistake. It really is true that there are no zeros of the quadratic differential in the complement of the trajectories mentioned. Thus they are rectangles, perhaps of infinite length, and this really happens on surfaces of infinite area. But on a surface of finite area, obviously there can be no infinite band, so the components of the complement are all rectangles (proving recurrence at the same time).

Still, I should have been more explicit. I should mention just what statement about complete Euclidean surfaces I am using:

A locally Euclidean surface with distinguished horizontal and vertical directions, with horizontal and vertical boundary, such that horizontal lines can be continued until they hit the boundary, and of finite area, is isometric to a rectangle.

Q At the left of Figure 5.3.8 (page 215) there are clearly two rectangles that have a zero of q' on their long edges. Since that zero is not a vertex of any of these rectangles, is what I see really half-planes (angle π, one for each rectangle with that zero in its boundary)? Is this correct?

A Yes, that is correct.