# Vector Calculus, Linear Algebra and Differential Forms: A Unified Approach 4th Edition, Second Printing

Complete List of Errata and Notes as of July 10, 2015

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Page 10 Second paragraph after the heading "What do we mean by rule?":

the function $M : [0, 1] \to \mathbb{R}$ that takes every number ...

not

the function $M : \mathbb{R} \to \mathbb{R}$ that takes every number ...

Also, "If the number has a 1, it changes all the digits after the first 1 into 0's and considers the result as a number in base 2"

Should be:

If the number written in base 3 must contain a 1, it changes every digit after the first 1 to 0, then changes every 2 to 1, and considers the result as a number in base 2.

(Following the former rule would change 0.2012 to 0.2010, which is not a number in base 2.)

Page 16 Second margin note, next-to-last paragraph: A relation is 1–1 if for all $y$ the set $\{x|(x, y) \in R\}$ has at most one element (not the set $\{y|(x, y) \in R\}$).

Page 18 Possible additions to Definitions 0.5.1 and 0.5.2:

If $X$ is unbounded above, $\sup X$ is defined to be $+\infty$.

If $X$ is unbounded below, $\inf X$ is defined to be $-\infty$.

Page 19 Third line: "digit of $b_n$", not "digit of $b_k$".

The next paragraph has been rewritten:

We claim that $b = \sup X$. Indeed, if there exists $y \in X$ with $y > b$, then there is a first $k$ such that the $k$th digit of $y$ differs from the $k$th digit of $b$. This contradicts our assumption that $b_k$ was the largest number (out of 10) such that $X \cap [b_k, a] \neq \emptyset$, since using the $k$th digit of $y$ would give a bigger

one. So $b$ is an upper bound. Now suppose that $b' < b$. If $b'$ is an upper bound for $X$, then $(b', a] \cap X = \emptyset$. Again there is a first $k$ such that the $k$th digit of $b$ differs from the $k$th digit of $b'$. Then $(b', a] \cap X \supset [b_k, a] \cap X \neq \emptyset$. Thus $b'$ is not an upper bound for $X$.

PAGE 68 Last margin note, $\vec{\mathbf{x}} \cdot \vec{\mathbf{e}}_i = x_i$, not $\vec{\mathbf{x}} \cdot \vec{\mathbf{e}}_i = x_1$

PAGE 70 The equation in the caption of Figure 1.4.3 should have absolute values:

$$|\vec{\mathbf{x}} \cdot \vec{\mathbf{y}}| = |\vec{\mathbf{x}}||\vec{\mathbf{y}}| \, |\cos \alpha|.$$

PAGE 75 Corollary 1.4.12 is out of place, since we have not defined what it means for a function of several variables to be continuous. In any case on page 99 we prove that a linear transformation from $\mathbb{R}^n \to \mathbb{R}^m$ is not just continuous but uniformly continuous.

PAGE 78 In Proposition 1.4.19, part 3, we use the expression "linearly independent", which hasn't been defined yet. We should have said "not collinear".

PAGE 81 In the table of correspondence, the indices for the sum at upper left should be the same for both $v$ and $w$: $v_i w_i$, not $v_j w_i$.

PAGE 91 First margin note: The second 0 should be bold:

$$\lim_{m \to \infty} (c_m \mathbf{a}_m) \neq \mathbf{0}.$$

PAGE 94 In Proposition 1.5.22, we are missing $(\mathbf{x})$ in several places: it should be

$$\lim_{\mathbf{x} \to \mathbf{x}_0} \mathbf{f}(\mathbf{x}) = \mathbf{a} \text{ exists} \quad \Longleftrightarrow \quad \text{each } \lim_{\mathbf{x} \to \mathbf{x}_0} f_i(\mathbf{x}) = a_i \text{ exists}$$

and

$$\lim_{\mathbf{x} \to \mathbf{x}_0} \mathbf{f}(\mathbf{x}) = \begin{pmatrix} \lim_{\mathbf{x} \to \mathbf{x}_0} f_1(\mathbf{x}) \\ \vdots \\ \lim_{\mathbf{x} \to \mathbf{x}_0} f_m(\mathbf{x}) \end{pmatrix}$$

PAGE 95 In Theorem 1.5.23, part 4, we should have defined the notation $(\mathbf{f} \cdot \mathbf{g})(\mathbf{x})$: Set $(\mathbf{f} \cdot \mathbf{g})(\mathbf{x}) \stackrel{\text{def}}{=} \mathbf{f}(\mathbf{x}) \cdot \mathbf{g}(\mathbf{x})$.

PAGE 96 Example 1.5.25: Equation 1.5.49 would be better as

$$f\begin{pmatrix} x \\ y \end{pmatrix} = \begin{cases} \dfrac{|y| e^{-|y|/x^2}}{x^2} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0,\ y \neq 0 \end{cases}$$

PAGE 99 Definition 1.5.31, third line: "for all $\mathbf{x}, \mathbf{y} \in X$", not "for all $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$".

PAGE 110 We should have specified that $C$ is a subset of $\mathbb{R}^n$. In Definition 1.6.6., $b \in C$ should be $\mathbf{b} \in C$, and "is called the maximum" should be "is called a maximum", since there could be other points at which the value of $f$ is $M$. Similarly, in Definition 1.6.8, $c$ should be $\mathbf{c}$ and "the minimum" should be "a minimum".

PAGE 117 Two lines before inequality 1.6.25, $b_j u^j$ should be $|b_j u^j|$, since $b_j$ and $u^j$ are complex numbers, so $b_j u^j$ isn't a distance.

PAGE 118 Two lines above equation 1.6.30: "of degree 1" (not "of degree 1 or 2"). But for real polynomials, "degree 1 or 2" in Corollary 1.6.15 is correct.

PAGE 128 In equation 1.7.23 and the line immediately after the equation, $\overrightarrow{D_i \mathbf{f}}$ should be $\overrightarrow{D_i \mathbf{f}}(\mathbf{a})$:

"$L(\vec{\mathbf{e}}_i) = \overrightarrow{D_i \mathbf{f}}(\mathbf{a})$, where $\overrightarrow{D_i \mathbf{f}}(\mathbf{a})$ is ... "

PAGE 129 First line after equation 1.7.27: "The first term is equation 1.7.4", not equation 1.7.35.

Two lines before equation 1.7.35, $\overrightarrow{D_i \mathbf{f}}$ should be $\overrightarrow{D_i \mathbf{f}}(\mathbf{a})$.

PAGE 130 First line: We don't need the triangle inequality here.

Three lines after equation 1.7.27, $\overrightarrow{D_i \mathbf{f}}$ should be $\overrightarrow{D_i \mathbf{f}}(\mathbf{a})$.

PAGE 135 In equation 1.7.52, the [0] at the end should be 0.

PAGE 137 In the last line of the proof of Proposition 17.19, two 0 should be [0], and one [0] should be 0: it should be

$$\lim_{H \to [0]} \frac{1}{|H|} \left| (A+H)^{-1} - A^{-1} + A^{-1} H A^{-1} \right| \leq \lim_{H \to [0]} 2|H||A^{-1}|^3 = 0.$$

PAGE 138 In Exercise 1.7.4, we should have defined the functions in parts b and c to be 0 at 0:

b. $f(x) = \begin{cases} x \ln |x| & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$ c. $f(x) = \begin{cases} x / \ln |x| & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$

In Exercise 1.7.10, part a, "linear" should be "affine". In part b, "nonlinear" should be "not affine".

PAGE 147 Exercise 1.8.6, part b: We should have said that $U$ is an open subset of $\mathbb{R}^3$.

PAGE 151 First line after equation 1.9.16: Theorem 1.5.23, part 5, not part 7

PAGE 156 Exercise 1.9: $S, T : \mathbb{R}^3 \to \mathbb{R}^3$, not $\mathbb{R}^4 \to \mathbb{R}^3$

PAGE 169 Theorem 2.2.2 presupposes that we know that the matrix $\widetilde{A}$ obtained by row reduction is unique. This is part 2 of Theorem 2.1.7, which is not proved until page 172. Theorem 2.2.2 should come after that proof.

PAGE 177 Immediately after Proposition 2.3.2: "if a matrix is square", not "if a matrix is invertible".

PAGE 179 In the bulleted list, $E_1$ should be $E_1(i, x)$; $E_2$ should be $E_2(i, j, x)$; and $E_3$ should be $E_3(i, j)$.

PAGE 202 Figure 2.5.2: The caption for the left side should be "The function $\dfrac{1}{x^2 + 1/10}$, between $x = -1$ and $x = 1$".

PAGE 210 Exercise 2.5.18 last line: $\in \mathbb{R}^{k+1}$, not $\in \mathbb{R}^k$.

Exercise 2.5.21 has an extraneous $H$: let $P_{\{\overline{\mathbf{v}}\}} H : \mathbb{R}^n \to \mathbb{R}^m$ should be let $P_{\{\overline{\mathbf{v}}\}} : \mathbb{R}^n \to \mathbb{R}^m$.

PAGE 214 Typo in equation 2.6.7: *dyr* should be *dy*.

PAGE 218 Equation 2.6.29 should have primes on the right:

$$[P_{\vec{\mathbf{v}}' \to \vec{\mathbf{e}}}] = [\vec{\mathbf{v}}'_1, \dots, \vec{\mathbf{v}}'_n].$$

PAGE 227 Second line after heading, "procedure for finding eigenvectors": "any $\vec{\mathbf{w}} \in \mathbb{C}^n$" should be "any nonzero $\vec{\mathbf{w}} \in \mathbb{C}^n$."

Also, in equation 2.7.21, there is an end parenthesis missing, before the first $\vec{\mathbf{w}}$ on the right side.

PAGE 242 In equation 2.8.45, $\mathbf{f}$ should be $\mathbf{F}$.

PAGE 243 The top line has an extraneous $Y$: $\mathbf{f}\begin{pmatrix} X \\ Y \end{pmatrix} Y = \begin{pmatrix} a \\ b \end{pmatrix}$ should be $\mathbf{f}\begin{pmatrix} X \\ Y \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix}$.

Line after inequality 2.8.53: $[\mathbf{D}\widetilde{f}\,]$ should be $[\widetilde{\mathbf{Df}}\,]$.

PAGE 247 In Example 2.8.16, shortly before equation 2.8.67, the parenthetical comment "remembering that since $A_0$ is diagonal, $A_0 h = h A_0$" is wrong. It should be "remembering that since $A_0$ is a scalar multiple of the identity, $A_0 H = H A_0$".

PAGE 249 Bottom of page: We neglected to include $D_2 D_1$ (we will see in Theorem 3.3.9 that $D_2 D_1 = D_1 D_2$). So the list should be

$$\sup |D_1 D_1 f_1| \le 3 = c_{1,1,1} \qquad \sup |D_1 D_1 f_2| = 0 = c_{2,1,1}$$
$$\sup |D_1 D_2 f_1| \le 1 = c_{1,2,1} \qquad \sup |D_1 D_2 f_2| = 0 = c_{2,2,1}$$

$$\sup|D_2D_1f_1| \le 1 = c_{1,1,2} \qquad \sup|D_2D_1f_2| = 0 = c_{2,1,2}$$
$$\sup|D_2D_2f_1| \le 1 = c_{1,2,2} \qquad \sup|D_2D_2f_2| = 2 = c_{2,2,2}.$$

PAGE 260 First line after Definition 2.10.1: "If a continuous function $f$ is monotone", not "if a function $f$ is monotone".

PAGE 261 In the displayed equation in the margin note, $f^{-1}$ should be $\mathbf{f}^{-1}$.

PAGE 267 Caption to Figure 2.10.8: $\overline{U_1}$, not $U_1$, in two places.

PAGE 268 In inequality 2.10.21, the third equality should be an inequality:

$$\le \sqrt{4\big((u_1-v_1)^2+(u_2-v_2)^2\big)+4\big((u_1-v_1)^2+(u_2-v_2)^2\big)}$$

PAGE 272 Second line part part of the Remarks: "every point in $W_0$", not "every point $W_0$".

PAGE 275 Example 2.10.18 proves that there is no differentiable inverse. It does not prove that there is no *non-differentiable* inverse.

PAGE 281 Last line of Exercise 2.28: "the inverse function at $\mathbf{f}\begin{pmatrix}0\\ \pi\end{pmatrix}$", not "the inverse function at $\begin{pmatrix}0\\ \pi\end{pmatrix}$".

PAGE 283 Exercise 2.38: for $n \ge 0$, not $n \ge 2$.

PAGE 286 margin note: 2nd paragraph, $n-k$, not $n-l$: "How then might we describe a point $\mathbf{z}$ in the graph of a function $\mathbf{f}:\mathbb{R}^k \to \mathbb{R}^{n-k}$?

PAGE 307 First line after Definition 3.2.1: "tangent line to a curve", not "tangent line to a plane".

PAGE 322 We should have added the following to Definition 3.3.14: If $\mathbf{f}:U \to \mathbb{R}^n$ is a $C^k$ function, its Taylor polynomial is the polynomial map $U \to \mathbb{R}^n$ whose coordinate functions are the Taylor polynomials of the coordinate functions of $\mathbf{f}$.

PAGE 345 Theorem 3.6.3: We should have said "if $\mathbf{x}_0 \in U$ is a local extremum ... " (adding word "local").

PAGE 347 Two lines after equation 3.6.10: "so $f$ has a local minimum" should be "so it has a local minimum".

PAGE 349 Line after equation 3.6.15: $t > 0$, not $t\mathbf{e}0$.

PAGE 353 First line: $[\mathbf{DF}(\mathbf{c})]$ should be $[\mathbf{D}F(\mathbf{c})]$.

PAGE 363 Three lines after equation 3.7.46: we should have written "of $f$ restricted to $X$": Moreover, the critical point $\begin{pmatrix}\mathbf{x}_0\\ \mathbf{y}_0\end{pmatrix}$ of $f$ restricted to $X$ has the same signature as the critical point $\mathbf{x}_0$ of $f\circ\widetilde{\mathbf{g}}$.

PAGE 365 Very end of Example 3.7.13, part 3: "the signature of the constrained critical point is $(0,2)$, so the critical point is a minimum" should be " ... so the critical point is a maximum".

PAGE 366 Line -4 of the remark: "no vector $\vec{\mathbf{v}}\in\mathbb{R}^2$" should be "no nonzero vector $\vec{\mathbf{v}}\in\mathbb{R}^2$".

PAGE 369 In Exercise 3.7.8, $a,b\geq 0$ should be $a,b>0$.

PAGE 375 Second line of equation 3.8.20: $(g(X)^2$ should be $(g(X))^2$.

PAGE 390 Exercise 3.8: We should have said that $M_1(m,n)$ is the subset of $\mathrm{Mat}\,(m,n)$ consisting of matrices of rank 1.

PAGE 409 We propose changing Exercise 4.1.20, to:

Let $a,b>0$ be real numbers, and let $T$ be the triangle defined by $x\geq 0$, $y\geq 0$, $x/a+y/b\leq 1$. Using upper and lower sums, compute the integral

$$\int_T x\,dx\,dy=\frac{a^2b}{6}.$$

*Hint*: For all $c\in$ , the *floor* $\lfloor c\rfloor$ is the largest integer $\leq c$. Show that

$$L_N(f)=\frac{1}{2^{2N}}\sum_{k=0}^{\lfloor 2^N a\rfloor}\ \sum_{j=0}^{\lfloor 2^N b\left(1-\frac{k+1}{2^N a}\right)\rfloor}\frac{k}{2^N}$$

and find the analogous formula for $U_N(f)$. In evaluating the double sum, note that the term $k/2^N$ does not depend on $j$, so the inner sum is just a product.

PAGE 427 Proof of Corollary 4.3.8: In thie first sentence, "number $A$ such that the number of cubes" should be "number $A$ such that for all $N$, the number of cubes". The sentence that starts three lines after formula 4.3.9 should read:

For any $C\in\mathcal{D}_N(\mathbb{R}^n)$ such that $C\cap X\neq\emptyset$, at most $2^N\epsilon+2$ cubes of $\mathcal{D}_N(\mathbb{R}^{n+1})$ that project to $C$ intersect $\Gamma_f$, hence $\Gamma_f$ is covered by at most $A2^{nN}(2^N\epsilon+2)$ cubes with total volume

$$\frac{1}{2^{(n+1)N}}A2^{nN}(2^N\epsilon+2),$$

($2^N\epsilon+2$, not $2^N\epsilon+1$. Note also the addition of "that project to $C$".)

PAGE 427 Corollary 4.3.9: $f:U\to\mathbb{R}$ not $f:U\to\mathbb{R}^n$

PAGE 428 3rd line of the proof at the top of the page: by Proposition 4.1.23, not Definition 4.1.23.

PAGE 430 Since Definition 4.4.1 is a definition, "and only if" is not needed.

PAGE 432 Example 4.4.5: By "in lowest terms" we should have said that we include the requirement that the denominator be positive, so that $f$ is well defined for negative numbers: $-\frac{1}{2}$ can be written only as $\frac{-1}{2}$; it cannot be written as $\frac{1}{-2}$.

PAGE 441 Margin note, last line: upper limit, not upper integral. Equation 4.5.10: the integral on the left should be over $P$, not over $\mathbb{R}^3$:

$$\int_P f\begin{pmatrix} x \\ y \\ z \end{pmatrix} |dx\,dy\,dz| = \int \left( \int \left( \int f\,dx \right) dy \right) dz.$$

PAGE 443 In Example 4.5.7 there are five 0 that should be $\mathbf{0}$; in the margin note for the example there are two more.

PAGE 460 Exercise 4.6.4: On the right side of the displayed equation, $f$ should be $p$.

PAGE 483 Two lines after equation 4.9.23: $\text{vol}_2(E(Q))$, not $\text{vol}(E(Q))$. More important, in the footnote, the left side of the displayed equation should be $\text{vol}_2(E(Q))$, not $E(Q)$.

PAGE 499 Proof of Theorem 4.11.2: In inequality 4.11.5, the nonstrict inequality can be strict; in the next line, the strict inequality should be nonstrict: $U(f) - L(f) \le \epsilon(1 + 2\,\text{vol}_n(B))$.

PAGE 505 In Example 4.11.11, we neglected to prove that the function $f$ defined in equation 4.11.43 is not L-integrable on $\mathbb{R}^n$ if $m \le n$. In the future this will be an exercise.

PAGE 505 Equation 4.11.45: "where $f_i(\mathbf{x}) = \dots$", not "where $f_i = \dots$".

PAGE 509 Theorem 4.11.19, line before equation 4.11.67: We should have said "then $f$ is L-integrable", not "then $f$ is integrable".

PAGE 511 We have made a few changes in the margin note.The big change is that $j = \infty$ and $k = \infty$ on the second line of the last formula have been changed to $j =$ and $k = 0$. It now reads:

Equation 4.11.74: To apply Fubini's theorem 4.11.20, we need to check that the function

$$f : \mathbb{R} \times \mathbb{R} \to \mathbb{R},\ f\begin{pmatrix} x \\ y \end{pmatrix} = e^{-(x^2+y^2)}$$

is L-integrable over $\mathbb{R}^2$. First note that $x \mapsto e^{-x^2}$ is L-integrable over $\mathbb{R}$ (see Definition 4.11.8, noting that $e^{-x^2} > 0$ and writing

$e^{-x^2} = \sum_{j=-\infty}^{\infty} e^{-x^2}\mathbf{1}_{[j,j+1)}$):

$$\sum_{-\infty}^{\infty}\int_{j}^{j+1} e^{-x^2}|dx| \le 2\sum_{j=0}^{\infty} e^{-j^2} < 2\sum_{j=0}^{\infty}\frac{1}{2^j} = 4.$$

Next, set

$$\sum_{j,k=-\infty}^{\infty} = \sum_{j=-\infty}^{\infty}\sum_{k=-\infty}^{\infty}$$

and write

$$\begin{aligned}
&\sum_{j,k=-\infty}^{\infty}\int_{j}^{j+1}\int_{k}^{k+1} e^{-(x^2+y^2)}|dx\,dy| \\
&\le 4\sum_{j=0}^{\infty}\sum_{k=0}^{\infty} e^{-(j^2+k^2)} \\
&= 4\sum_{j=0}^{\infty} e^{-j^2}\left(\sum_{k=0}^{\infty} e^{-k^2}\right) \\
&< 16.
\end{aligned}$$

PAGE 514 Line 5 after Definition 4.11.26: "the function $t \mapsto te^{-st/2}$ is L-integrable", not "the function $t \mapsto e^{-st/2}$ is L-integrable." Four errors in equation 4.11.94 (three missing $t$ and missing absolute values). It should be:

$$\left|\frac{e^{-ht}-1}{h}\right| \le te^{|ht|} \le te^{st/2}, \text{ so } \left|e^{-st}f(t)\frac{e^{-ht}-1}{h}\right| \le te^{-st/2}|f(t)|.$$

PAGE 516 Exercise 4.11.16: The sum should be $\sum_{k=0}^{\infty}$, not $\sum_{k=1}^{\infty}$.

PAGE 533 Line −5: "area of a surface $S$ parametrized", not "area of a surface parametrized".

PAGE 535 The 5th line of first margin note should be

$$\int_U f(\Phi(\mathbf{u}))\,|\det[\mathbf{D}\Phi(\mathbf{u})]|\,|d^n\mathbf{u}|.$$

(Remove the first open parenthesis and close the absolute values around the det.)

PAGE 540 First line of equation 5.3.51, on the right side: $\int_{B_R(\mathbf{0})}$, not $\int_{B_{\mathbf{0}}(\mathbf{0})}$.

PAGE 541 Line 5: "$(n+1)$-dimensional ball", not "$(n+1)$-dimensional sphere".

PAGE 543 In Exercise 5.3.15, the description of the map $\gamma$ should be

$$\gamma : \begin{pmatrix} \theta \\ \varphi \\ \psi \end{pmatrix} \mapsto \dots \quad \text{or} \quad \gamma \begin{pmatrix} \theta \\ \varphi \\ \psi \end{pmatrix} = \dots$$

PAGE 547 Equation 5.4.19 should have squares on the cos and sin:

$$U_r = \left\{ \begin{pmatrix} \rho \\ \theta \end{pmatrix} \;\middle|\; \rho \le r - \frac{r^3}{6} \left( a \cos^2 \theta + b \sin^2 \theta \right)^2 + o(r^3) \right\}.$$

PAGE 548 Equation 5.4.23: In the second line, the upper limit of integration for the second integral should include a square:

$$\int_0^{r - \frac{r^3}{6}(a \cos^2 \theta + b \sin^2 \theta)^2 + o(r^3)} .$$

In the third line, $b \sin^2 t$ should be $b \sin^2 \theta$.

PAGE 551 To be consistent with our usual notation, $|d\mathbf{x}|^2$ should be $|d^n\mathbf{x}|$ (in Theorem 5.4.6, the line before the g, and the discussion following it).

PAGE 569 In equation 6.1.42 and in the line after it, there are two $\mathbf{v}_1$ that should be $\vec{\mathbf{v}}_1$.

PAGE 579 Example 6.3.6: "(i.e., $a$, $b$, and $c$ are all nonzero)" not "(i.e., at least one of $a$, $b$, and $c$ does not vanish)".

PAGE 580 Example 6.3.7, first line: $X$ should be $S$

PAGE 583 Exercise 6.3.13, part a: The maps should be continuous: "Find two continuous mappings ... "

PAGE 584 Note that if $T$ in Definition 6.4.1 is not injective, then it is neither orientation preserving nor orientation reversing, since in that case $T(\vec{\mathbf{e}}_1), \dots, T(\vec{\mathbf{e}}_k)$ is not a basis of $V$.

PAGE 584 Definition 6.4.2: "$\gamma : (U - X) \to M$ is a strict parametrization" is wrong. That sentence and the sentence before it can be replaced by "Let $\gamma : U \to \mathbb{R}^m$ parametrize $M$ as described in Definition 5.2.3, so that the set $X$ of "trouble spots" satisfies all conditions of that definition; in particular, $X$ has $k$-dimensional volume 0."

PAGE 599 Equation 6.5.14: The second integral is missing an end parenthesis. It should be

$$\int_{[a,b]} W_{\vec{F}}\Big( P_{\gamma(u)}\big( \overrightarrow{D_1\gamma(u)} \big) \Big) |du|$$

PAGE 608 The first sentence after Definition 6.6.2 might be clearer as "Part 1 just says that $M$ is a manifold; it is there to set up the notation for part 2."

PAGE 611 The second equation in 6.6.8 should be

$$g(\mathbf{y}) = \alpha_i(\mathbf{x} + \vec{\mathbf{v}}_i) - \alpha_i(\mathbf{y}),$$

to satisfy the requirement $g \geq 0$ of Definition 6.6.2 (see formula 6.6.7).

PAGE 611 The last sentence of the paragraph beginning "Condition 1 of Definition 6.6.6" should be

"This follows from Proposition 5.2.2; indeed this set is a subset of finitely many affine subspaces of dimension $k-2$. "

PAGE 613 Line immediately before equation 6.6.15: The matrix $B$ should be $n \times k$, not $k \times n$.

PAGE 619 Exercise 6.6.4, part c: The matrix $B$ should be $n \times k$, not $k \times n$.

PAGE 624 First line of equation 6.7.19: $d\omega_n$ should be $\mathbf{d}\omega_n$.

PAGE 635 In Theorem 6.9.2 the hypothesis "compact" is not needed, since by Definition 6.6.6 a piece-with-boundary is compact.

PAGE 640 In the line immediately after equation 6.9.28, $\epsilon^{-k}(\epsilon^{k+1}) = \epsilon$, not $\epsilon^{-k}(e^{k+1}) = \epsilon$ (the $e$ should be $\epsilon$).

PAGE 648 Second margin note: The reference should be to equation 6.10.25 not 6.10.26.

PAGE 657 In equations 6.1.24 and 6.11.25, $\mathbf{y} \overrightarrow{-} \mathbf{x}$ should be $\mathbf{x} \overrightarrow{-} \mathbf{y}$.

PAGE 658 The cross product in the caption to Figure 6.11.4 should be $h\vec{\mathbf{j}}(\mathbf{y}) \times \frac{\overrightarrow{\mathbf{x}-\mathbf{y}}}{|\mathbf{x}-\mathbf{y}|^3}$.

PAGE 659 In equation 6.11.36 and in the last two displayed equations in the margin, the integrals should be over $\Gamma$ (the wire), not over $W$.

PAGE 669 Line after equation 6.12.4: first entry, not third entry.

PAGE 671 Equation 6.12.10: $t_1\vec{\mathbf{v}}_1 + \cdots + t_k\vec{\mathbf{v}}_k$, not $t_1\vec{\mathbf{v}}_1 + \cdots t_k\vec{\mathbf{v}}_k$.

PAGE 672 In the line immediately after formula 6.22.14, $k$ not $n$: "a direct basis for $CP_{\mathbf{x}}(\vec{\mathbf{v}}_1, \ldots, \vec{\mathbf{v}}_k)$".

PAGE 673 Last line of equation 6.12.20: the bracketed explanation should be "$k$ vectors in $\mathbb{R}^n$: $\overrightarrow{D_0\gamma}, \overrightarrow{D_1\gamma} \ldots, \overrightarrow{D_{k-1}\gamma}$"(i.e., the last vector should not

be $\overrightarrow{D_k\gamma}$. In line $-5$ of the last margin note, "to evaluate $f$" should be "to evaluate $\varphi$".

PAGE 674 Line after equation 6.12.25: "we evaluate $\mathbf{c}\varphi$", not "we evaluate $\varphi$".

In the margin note, "The second $t_0$ corresponds to $t_0^{k-1}$ in equation 6.12.25" should be "The second $t_0$ corresponds to $t_0^{k-1}$ in equation 6.12.24".

In equtions 6.12.21 and 6.12.22, $\mathbf{x}_0$ should be $\mathbf{x}$.

PAGE 676 There should be a $1/c^2$ in front of the $\vec{\nabla} D_t \vec{A}$ in equation 6.12.42.

PAGE 681 Exercise 6.26 should read:

Show that the electromagnetic field of a charge $q$ moving in the direction of the $x$-axis at constant speed $v$ is

$$\vec{\mathbf{E}} = \frac{q\gamma}{4\pi\Big((\gamma x - \gamma v t)^2 + y^2 + z^2\Big)^{3/2}} \begin{bmatrix} x - vt \\ y \\ z \end{bmatrix}$$

$$\vec{\mathbf{B}} = \frac{v}{c}\ \frac{q\gamma}{4\pi\Big((\gamma x - \gamma v t)^2 + y^2 + z^2\Big)^{3/2}} \begin{bmatrix} 0 \\ -z \\ y \end{bmatrix}, \quad \text{where } \gamma = \frac{1}{\sqrt{1 - v^2/c^2}}.$$

PAGE 682 First margin note: The right side of the displayed equation should have a minus sign:

$$\mathbf{d}\left(\frac{1}{r}\right) = -W_{\vec{\mathbf{r}}/r^3}$$

PAGE 701 Two lines before the exercises:

$$|\mathbf{y} - \mathbf{a}_0| \le 2|\vec{\mathbf{h}}_0|, \quad \text{not} \quad |\mathbf{y} - \mathbf{a}_0| \le 2\vec{\mathbf{h}}_0.$$

PAGE 702 Exercise A5.1: In the second line, $0 < \alpha$, not $0 \le \alpha$.

PAGE 710 Two lines before the remark, " $\mathbf{F}\begin{pmatrix}\mathbf{x}\\ \mathbf{y}\end{pmatrix}$ implicitly defines $\mathbf{x}$ " should be " $\mathbf{F}\begin{pmatrix}\mathbf{x}\\ \mathbf{y}\end{pmatrix} = \mathbf{0}$ implicitly defines $\mathbf{x}$ ".

PAGE 712 Last sentence of the proof: "the expression on the first line is symmetric", not "the expression on the right is symmetric".

PAGE 719 Equation A11.20: in the second line, $\in$ should be $\subset$.

PAGE 731 Three lines before equation A15.3: "since $Z$, as a function of $X$ and $Y$, starts with quadratic terms", not "since $Z$ as a function of $X$ and $Y$ that starts with quadratic terms".

PAGE 744 First line in equation A18.9: The sum is over $P \in \mathcal{P}_{N''}$, not over $P \in \mathcal{P}''_N$.

PAGE 745 In the last line, "i.e., $k$ is the first column" should be "i.e., $\mathbf{a}_k$ is the first column".

PAGE 746 Two lines before equation A19.4: $A_{1,i}$ should be $A_{i,1}$. In the same line, $\widetilde{A}_{1,i}$ should be $\widetilde{A}_{i,1}$, as it should in equation A19.4. Recall that $A_{i,j}$ is $A$ with the $i$th row and $j$th column removed.

PAGE 749 Equation A19.15: In the first matrix, $\widetilde{B}$ should be $\widetilde{Q}$.

PAGE 751 The right side of inequality A20.4 needs a factor of 2 in the denominator: $K\frac{\sqrt{n}}{2\cdot 2^N}$

PAGE 754 Margin note: The third line of the equation that starts $\operatorname{vol}_n \Phi(C)$ has an extra $C$ that shouldn't be there:

$$= (1+\epsilon)^n |\det[\mathbf{D}\Phi(\mathbf{z}_C)]C| \operatorname{vol}_n C.$$

should be

$$= (1+\epsilon)^n |\det[\mathbf{D}\Phi(\mathbf{z}_C)]| \operatorname{vol}_n C.$$

PAGE 760 6th line: Using our current notation, $L_{\chi_{A_k}}$ should be $L_{\mathbf{1}_{A_k}}$. In the second margin note, 3rd line, $\int_Q \inf(f_k(\mathbf{x}), K)$ should be $\inf(f_k(\mathbf{x}), K)$

PAGE 781 When reprinting Theorem 6.7.2, we omitted part 3: The exterior derivative of a constant form is 0.

PAGE 788 Three lines from the bottom: $\mathbf{df}^* = \mathbf{f}^*\mathbf{d}$, not $d\mathbf{f}^* = \mathbf{f}^* d$.

PAGE 767 In the next-to-last line of equation A22.49, $\mathbb{R}^n$ is missing from the second integral; it should be

$$\int_{\mathbb{R}^n} |f_{k,i}(\mathbf{x})| \, |d^n\mathbf{x}|$$

PAGE 767 Margin note: Right before the first displayed equation, the definition of $h_l$ should be $h_l = \sum_{j=1}^{l} g_{j,l-j+1}$, not $h_l = \sum_{j=1}^{l} g_{j,l-j}$. Thus $h_1 = g_{1,1}$, $h_2 = g_{1,2} + g_{2,1}$, $h_3 = g_{1,3} + g_{2,2} + g_{3,1}$, and so on.

PAGE 772 One line after inequality A22.71: "by equations A22.70 and A22.70" should be "by equations A22.70 and A22.71".

PAGE 781 When reprinting Theorem 6.7.2, we omitted part 3: The exterior derivative of a constant form is 0.

PAGE 783 Equation A24.8: The first sup is over $I \in \mathcal{I}_n^2$, not $I \in \mathcal{I}_n^{k+2}$.

PAGE 788 Three lines from the bottom: $\mathbf{df}^* = \mathbf{f}^*\mathbf{d}$, not $d\mathbf{f}^* = \mathbf{f}^*d$.

PAGE 795 In equation A26.17, the domain of the integral immediately following the equality marked 4 is ambiguous. It would be better to write it as $W_i \cap \partial Z_i$.

PAGE 797 In equation A26.33, $|d^k\mathbf{x}|$ should be $|d^{k-1}\mathbf{x}|$. Equation A26.34 should be

$$\int_W |d^{k-1}\mathbf{w}| \underbrace{=}_{\text{Def. 5.3.1}} \int_W |d^{k-1}\mathbf{x}|\Big(P_{\delta(\mathbf{w})}(D_1\delta(\mathbf{w}),\dots,D_{k-1}\delta(\mathbf{w})\Big)|d^{k-1}\mathbf{w}|$$

The last four lines (beginning with "The integral") should be replaced by

The integral

$$\int_W |d^{k-1}\mathbf{w}| \underbrace{=}_{\text{Def. 5.3.1}} \int_W \underbrace{|d^{k-1}\mathbf{x}|\Big(P_{\delta(\mathbf{w})}(D_1\delta(\mathbf{w}),\dots,D_{k-1}\delta(\mathbf{w})\Big)}_{\text{this function of } \mathbf{w} \text{ is the dominating function}} |d^{k-1}\mathbf{w}| \quad A26.34$$

is finite by the hypothesis that $\partial^s_M X$ has finite $(k-1)$-volume, and we can use the "dominating function" above to prove that

$$\lim_{\epsilon\to 0}\int_{W_\epsilon} |d^{k-1}\mathbf{w}| = \int_W |d^{k-1}\mathbf{w}|.$$

PAGE 798 Margin note: "Two lines in $\mathbb{R}^3$ are never transversal" should be "Two intersecting lines in $\mathbb{R}^3$ (in particular, two 1-dimensional vector subspaces) are never transversal".

PAGE 800 In three places (A26.42, A26.43, and the line before A26.43) $\begin{bmatrix} I_k \\ [\mathbf{Df}(g(\mathbf{y}))] \end{bmatrix}$ should be $\begin{bmatrix} I_k \\ [\mathbf{Df}(\mathbf{x})] \end{bmatrix}$.

PAGE 801 First line after A26.44: "if at least", not "if and only if at least".

PAGE 802 One line before equation A26.45, "of radius $|[\mathbf{D}f(\mathbf{x})]|$" should be "of radius $|[\mathbf{D}f(\mathbf{x})]|r$".

## NONMATHEMATICAL TYPOS

PAGE 15 The sentence after Definition 0.4.12 is missing a period.

PAGE 44 Figure 1.2.1, second line of caption: "the entries $a_{i,k}$ of the matrix $A$", not "the entries of the $a_{i,k}$ of the matrix $A$".

PAGE 74 First line after equation 1.4.29: the word "that" is superfluous. The sentence should read: "We will consider the rows of $A$ ... ".

PAGE 67 Exercise 1.3.15: "Prove part 1 of Theorem 1.3.4", not "Prove theorem part 1 of Theorem 1.3.4".

PAGE 80 In the title of Proposition 1.4.20, the end parenthesis is missing.

PAGE 102 First line after equation 1.5.67: Proposition 1.4.11, not Proposition 1.4.11 b

PAGE 113 Three lines after formula 1.6.18: The solutions are presented in Appendix A.2, not in Section 0.7.

PAGE 116 2 lines before Figure 1.6.9: there is an extra absolute value; it should be $\rho < |b_0/b_j|^{1/j}$, not $\rho < |b_0/b_j|^{1/j}|$.

PAGES 117 AND 118 In Corollary 1.6.14 and in equation 1.6.31, the dots "$\cdot \dots \cdot$ would be better written "$\cdots$".

PAGE 140 Top margin note: "writing $\mathbf{f}$ and $\mathbf{g}$ as $\vec{\mathbf{f}}$ and $\vec{\mathbf{g}}$", not " ... in as $\vec{\mathbf{f}}$ and $\vec{\mathbf{g}}$"

PAGE 143 middle of page, part 7: "We do not need to prove", not "We do not need prove".

PAGE 182 Four lines from the bottom: "makes it is easy to analyze" should be "makes it easy to analyze".

PAGE 192 In the second line and in the fourth line, a comma is missing: $\vec{\mathbf{v}}_1, \dots \vec{\mathbf{v}}_k$ should be $\vec{\mathbf{v}}_1, \dots, \vec{\mathbf{v}}_k$.

PAGE 202 Comma missing two lines after equation 2.5.13: $p(0), \dots p(k)$ should be $p(0), \dots, p(k)$.

PAGE 219 Equations 2.6.33 and 2.6.34: In the vectors the entry $a_k, j$ should be $a_{k,j}$.

PAGE 229 Comma missing one line before Theorem 2.7.6:
"for $i = 1, \dots n$" should be "for $i = 1, \dots, n$".

PAGE 232 Comma missing in Exercise 2.7.4, first line: $\lambda_1, \dots, \lambda_n$, not $\lambda_1, \dots \lambda_n$.

PAGE 242 In equation 2.8.45, $\mathbf{F}$, not $\mathbf{f}$.

PAGE 254 Two lines after equation 2.9.2: "But once it starts, it is so fast", not "But once it starts, fast"

PAGE 260 Four lines from the bottom, there should be no comma after "The equation $1 = 2x + \sin x$".

PAGE 261 Lower margin note: $\mathbf{f}^{-1}$ not $f^{-1}$.

PAGE 273 Example 2.10.16, 3rd line: There should be a comma between $n = 3$ and $m = 2$.

PAGE 292 Line 3: "it does not represent", not "it does not represents".

PAGE 297 In next to last line, remove extra parenthesis: $[\mathbf{DF}(\mathbf{x})]$, not $[\mathbf{DF}(\mathbf{x}))]$

PAGE 306 Exercise 3.1.21, part b: a space is needed before "is a smooth curve".

PAGE 308 Second line of Example 3.2.3: $S \subset \mathbb{R}^3$, not $S \subset R^3$.

PAGE 368 First margin note: "be be impressed" should be "be impressed".

PAGE 373 Caption to Figure 3.8.2: At the end of the second paragraph, "at f w $\begin{pmatrix} 0 \\ 1/2 \end{pmatrix}$" should be "at $\begin{pmatrix} 0 \\ 1/2 \end{pmatrix}$".

PAGE 409 In part b of Exercise 4.1.18, an end parenthesis is in the wrong place: $f(x_{i+1}^2) - f(x_i^2)$, not $f\big((x_{i+1})^2\big) - f(x_i^2)$.

PAGE 431 In the last margin note, "integration" is misspelled, first as "intergration", then as "itegration".

PAGE 446 Equation 4.5.31: there should be no comma at the end of the first line.

PAGE 472 Definition 4.8.13: an $n \times n$ matrix, not "a $n \times n$ matrix". Two lines after the definition: matrices, not matrixes.

PAGE 476 There is an extra "is" in part c of Exercise 4.8.2; it should be "For each $n$, for what values of $a$ and $b$ is the matrix in part b not invertible?"

PAGE 482 in equation 4.9.21, the dots should be raised:
$[T] = E_k E_{k-1} \cdots E_1$.

PAGE 518 In Exercise 4.25 the word "by" is missing; "defined by the inequalities".

PAGE 531 Line after Theorem 5.2.10: The theorem is proved in Appendix A23, not Appendix 23.

PAGE 558 Six lines from the bottom: "are actually two instances", not "but are actually two instances"

PAGE 568 Margin note at bottom of page: The word "than" is missing. It should be "quite a bit harder than parts 1 and 3".

PAGE 569 Line immediately after equation 6.1.43: "we ought to specify", not "we ought specify".

PAGE 592 Line −2: "an integral", not "a integral".

PAGE 619 A comma is missing from the dislayed equation in Exercise 6.6.3: $\Omega(\vec{\mathbf{v}}_1, \dots, \vec{\mathbf{v}}_{n-1})$, not $\Omega(\vec{\mathbf{v}}_1, \dots \vec{\mathbf{v}}_{n-1})$.

PAGE 632 Equation 6.8.19, left side of first line: there is an extra end parenthesis; "$(\vec{\mathbf{v}}_1 \times \vec{\mathbf{v}}_2))$" should be "$(\vec{\mathbf{v}}_1 \times \vec{\mathbf{v}}_2)$".

PAGE 648 Example 6.10.8: Hiero, not Creon

PAGE 654 Two lines after equation 6.11.15: "about about" should be "about".

PAGE 670 First line of Example 6.12.3: given in equation 6.7.17, not given in Example 6.7.17.

Last line of proof of Proposition 6.12.2: "$(n-1)$-form", not "$n-1$-form".

PAGE 676 First margin note: $df$ should be $\mathbf{d}f$.

PAGE 693 First line: $\mathbf{x}_m, x_{m+1}$ should be $\mathbf{x}_m, \mathbf{x}_{m+1}$.

PAGE 753 An end parenthesis is missing from the caption to Figure A20.4: the point $\Phi^{-1}([\mathbf{D}\Phi(\mathbf{0})]\mathbf{x})$, not the point $\Phi^{-1}([\mathbf{D}\Phi(\mathbf{0})]\mathbf{x}$.