Appendix C9
Sullivan's dictionary

Sullivan's dictionary pairs statements about 1-dimensional complex dynamics (the subject of Chapter 9 and 10) and statements about 3-dimensional hyperbolic geometry (Volumes 3 and 4). For instance:

Complex dynamics	**Hyperbolic geometry**
Julia set	limit set
non-attracting cycles	non-elliptic fixed points
no wandering domain	Ahlfors finiteness
tuning	Klein-Maskit combination
topological characterization of rational functions	Thurston's hyperbolization theorem for Haken manifolds
parabolic blow-ups	geometric limits
matings	double limit theorem
Yoccoz inequality	compactness of Bers slices

Attempts (and failures) to work out various aspects of this dictionary have driven the enormous expansion of both subjects since 1980.

In some cases, the constructions on the two sides look similar although the proofs are different; for instance, the density of fixed points in the limit set for a Kleinian group is similar to the density of repelling cycles in the Julia set, but the proof of the former using the convex hull of the limit set has no known analogue for the Julia set.

In other cases the proofs are amazingly similar although the statements sound unrelated. For instance, the proof of Sullivan's no wandering domain theorem was modeled on that of the Ahlfors finiteness theorem; the proof of Thurston's theorem on the topological characterization of rational functions was modeled on that of his hyperbolization theorem for Haken manifolds.

There is a glaring gap in the dictionary: no known concept on the dynamical side corresponds to hyperbolic space $\mathbb{H}^3$. Thurston claimed that this is the greatest weakness of the theory.

In this appendix we explore three correspondences: Sullivan's no wandering domain theorem and Ahlfors finiteness; the Yoccoz inequality and compactness of Bers slices; and matings and the double limit theorem.

C9.1 Sullivan's no wandering domain theorem

Sullivan's no wandering domain theorem solved the main open problem in complex dynamics since the work of Fatou and Julia around 1920. It