Student Solution Manual

Vector Calculus, Linear Algebra, and Differential Forms: A Unified Approach, 4th Edition

Notes and Errata

Complete as of July 10, 2015

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New errata are found on pages 75, 158, 186, 239, 242, 243, 247, and 248.

Errors

Page 1 In Solution 0.4.13, $-1 \leq x < 0$ should be $-1 < x < 0$.

Page 2 Solution 0.5.1: Three lines from the end of the solution the inequality for $p(-A)$ should be

$$p(-A) \leq (-A)^d + (A-1)A^{d-1} < 0.$$

Page 15 Solution 1.4.19, part b should be " … which tends to $\pi/2$ as $n \to \infty$."

Page 21 Solution 1.5.21: The solution to part d should be

When $0 < x^2 + 2y^2 < 1$,

$$0 > (x^2+y^2)\ln(x^2+2y^2) \geq (x^2+y^2)\ln\big((x^2+y^2)\big),$$

which tends to 0 using the equation in the margin. So if we choose $f\begin{pmatrix}0\\0\end{pmatrix} = 0$, then f is continuous.

Page 29 Solution 1.9.1, last displayed equation:

$$8\frac{(x^2+y^2)^{5/2}}{(x^2+y^2)^2} = 8\sqrt{x^2+y^2}, \quad \text{not } = \sqrt{x^2+y^2}.$$

Solution 1.9.3, part a: The equation should be

$$\lim_{\vec{\mathbf{h}}\to\mathbf{0}} \frac{\dfrac{\sin(h_1^2h_2^2)}{h_1^2+h_2^2} - ah_1 - bh_2}{(h_1^2+h_2^2)^{1/2}} = 0.$$

Page 30 Solution 1.9.3, part c: This is wrong, and should be replaced by

Since $D_1 f(0) = 0$ and $D_2 f(0) = 0$, we see that f is differentiable at the origin if and only if

$$\lim_{|\mathbf{h}|\to 0} \frac{\sin(h_2^2 h_2^2)}{(h_1^2 + h_2^2)(h_1^2 + h_2^2)^{1/2}} = 0,$$

and this is indeed true, since

$$\left|\sin(h_2^2 h_2^2)\right| \le h_1^2 h_2^2 \le \frac{1}{4}\left(h_1^2 + h_2^2\right)^2 \tag{1}$$

and

$$\lim_{|\mathbf{h}|\to 0} \frac{1}{4} \frac{(h_1^2 + h_2^2)^2}{(h_1^2 + h_2^2)^{3/2}} = \frac{1}{4} \lim_{|\mathbf{h}|\to 0} (h_1^2 + h_2^2)^{1/2} = 0.$$

Equation 1, first inequality: For any x, we have $|\sin x| \le |x|$. The secod inequality follows from

$$0 \le (x-y)^2 = x^2 - 2xy + y^2,$$

so for any $x, y \in \mathbb{R}$ we have

$$|xy| \le \frac{1}{2}(x^2 + y^2).$$

PAGE 40 Solution 2.2.7, part a: In the displayed equations giving values for x, y, and x, the second x should be z:

$$z = \frac{1+b}{2+a+b}.$$

PAGE 43 Solution 2.3.3: In the second paragraph of part a, "the system of m equations in m variables" should be "the system of n equations in m variables". Later in the same paragraph, "we must have $n \le m$" should be "we must have $m \le n$" and "if $n < m$" should be "if $m < n$".

PAGE 50 Solution 2.5.9: $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ row reduces to $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$.

PAGE 50 The solution to part b of Exercise 2.5.11 is wrong. Here is a correction:

By row operations, we can bring the matrix B to

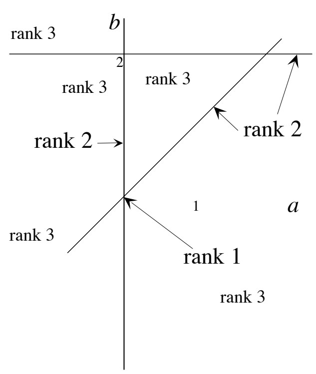
$$\begin{bmatrix} 1 & 2 & a \\ 0 & b & ab-a \\ 0 & 2a-b & a^2-a \end{bmatrix}.$$

We now separate the case $b \ne 0$ and $b = 0$.

- If $b = 0$, the matrix is $\begin{bmatrix} 1 & 2 & a \\ 0 & 0 & -a \\ 0 & 2a & a^2-a \end{bmatrix}$, which has rank 3 unless $a = 0$, in which case it has rank 1. (Of course, since $n = 3$, rank 3 corresponds to $\dim \ker = 0$, and rank 1 corresponds to $\dim \ker = 2$.)

- If $b \ne 0$, then we can do further row operations to bring the matrix to the form

$$\begin{bmatrix} 1 & 2 & a \\ 0 & b & ab-a \\ 0 & 0 & a^2 - a - \frac{2a-b}{b}(ab-a) \end{bmatrix}.$$



CORRECTED FIGURE

Solution 2.5.11, part b. On the a-axis, on the line $a = b$, and on the line $b = 2$, the image of B has dimension 2, i.e., its kernel has dimension 1. At the origin the rank is 1 and the dimension of the kernel is 2. Elsewhere, the kernel has dimension 0 and the rank is 3.

The third entry in the third row factors as $\frac{a}{b}(a-b)(2-b)$, so we have rank 2 (and $\dim\ker = 1$) if $a = 0$ or $a = b$ or $b = 2$. Otherwise we have rank 3 (and $\dim\ker = 0$).

PAGE 53 Solution 2.5.17, first line page 53: $\mathbb{R}^{2n+2}$, not R^{2n+2}

PAGE 57 Solution 2.7.5. The recursive formula for the b_n is

$$\begin{bmatrix} b_n \\ b_{n+1} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} b_{n-1} \\ b_n \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix}^n \begin{bmatrix} b_0 \\ b_1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix}^n \begin{bmatrix} 1 \\ 1 \end{bmatrix};$$

in the manual, the power n was missing on the matrix $\begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix}$. A basis of eigenvectors is

$$\begin{bmatrix} 1 \\ 1+\sqrt{2} \end{bmatrix}, \quad \begin{bmatrix} 1 \\ 1-\sqrt{2} \end{bmatrix}.$$

PAGE 68 Solution 2.10.9: The derivative given is wrong. It is

$$[\mathbf{D}F(\mathbf{0})] = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}.$$

PAGE 71 Solution 2.5, part a, after the displayed equation: "add multiples of the last m rows to the first n" (not to the first m).

PAGE 75 Solution 2.25: The solution to part b should read

The inverse of $[\mathbf{D}F(\mathbf{x})]$ is

$$[\mathbf{D}F(\mathbf{x})]^{-1} = \frac{1}{4xy-1} \begin{bmatrix} 2y & 1 \\ 1 & 2x \end{bmatrix} \quad \text{which at } \mathbf{x}_0 = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \text{ is } \quad \frac{1}{23} \begin{bmatrix} 6 & 1 \\ 1 & 4 \end{bmatrix},$$

so we have

$$\vec{\mathbf{h}}_0 = -\frac{1}{23} \begin{bmatrix} 6 & 1 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} +5/23 \\ -3/23 \end{bmatrix}, \quad \text{so} \quad \mathbf{x}_1 = \begin{bmatrix} 2+5/23 \\ 3-3/23 \end{bmatrix}.$$

The length of $[\mathbf{D}F(\mathbf{x}_0)]^{-1}$ is $\sqrt{54}/23$, so

$$|F(\mathbf{x}_0)| \ \left|[\mathbf{D}F(\mathbf{x}_0)]^{-1}\right|^2 \ M = \sqrt{2} \cdot \frac{54}{23^2} \cdot 2 \approx 0.289 < \frac{1}{2},$$

so Newton's method converges, to a point of the disc of radius

$$|\vec{\mathbf{h}}_0| = \frac{\sqrt{34}}{23} \approx .25 \quad \text{around } \mathbf{x}_1.$$

PAGE 78 Solution 2.33: The x in this solution should be $\mathbf{x}$.

PAGE 81 Solution 3.1.1: The drivative of $F\begin{pmatrix} x \\ y \\ z \end{pmatrix} = x^2+y^2+z^2-1$, not the derivative of $F\begin{pmatrix} x \\ y \end{pmatrix} = x^2+y^2+z^2-1$.

Solution 3.1.5, 3 lines before the end of part a: $y = -x^{2/3}$, not $y = x^{2/3}$

Solution 3.1.7, last displayed equation on the page: $\begin{pmatrix} 1/2 \\ \pm 1/\sqrt{3} \\ z \end{pmatrix}$ (in the second entry, $1/\sqrt{3}$, not $1\sqrt{3}$).

PAGE 82 Solution 3.1.7: "A the points $\begin{pmatrix} 1/2 \\ +1/\sqrt{3} \\ z \end{pmatrix}$, $\begin{pmatrix} 1/2 \\ -1/\sqrt{3} \\ z \end{pmatrix}$".

Solution 3.1.11, part b: $x = \dfrac{y^2}{z}$, not $x = \dfrac{y^2}{x}$

PAGE 89 Solution 3.2.5: In the second and third equations following "which expand to", aA^2 should be a^2A and bB^2 should be b^2B. The correct value for $\mathbf{q}$ is $\mathbf{q} = \begin{pmatrix} a/2 \\ b/2 \\ 0 \end{pmatrix}$, and the volume of the tetrahedron is

$$\frac{1}{6}\left|\det \begin{bmatrix} a & 0 & a/2 \\ 0 & b & b/2 \\ a^2A & b^2B & 0 \end{bmatrix}\right| = \frac{ab}{12}(a^2A + b^2B).$$

PAGE 105 Solution 3.6.7: In the equation for $D_2^2 f$, $4y^2$ should be $4y^4$:

$$D_2^2 f \begin{pmatrix} x \\ y \end{pmatrix} = (2 - 2x^2 - 10y^2 + 4x^2y^2 + 4y^4)e^{(x^2-y^2)}$$

PAGE 107 Solution 3.7.5, last line: The total volume is $4\sqrt{3}$, not $4\sqrt{2}$.

PAGE 139 Part a of Solution 4.5.13 would be better as

a. If $D_2(D_1(f))$ and $D_1(D_2(f))$ both exist and are continuous on U, and if there exists $\epsilon > 0$ such that

$$D_2(D_1(f))(\mathbf{a}) - D_1(D_2(f))(\mathbf{a}) > \epsilon, \tag{1}$$

then there exists $\delta > 0$ such that on the square

$$S = \left\{ \begin{pmatrix} x \\ y \end{pmatrix} \in \mathbb{R}^2 \mid a_1 \le x \le a_1 + \delta, a_2 \le y \le a_2 + \delta \right\}$$

$D_2(D_1(f)) - D_1(D_2(f)) > \epsilon$. In particular,

$$\int_S D_2(D_1(f)) - D_1(D_2(f))\,|dx\,dy| > \delta^2\epsilon > 0.$$

Note that this uses the continuity of the second partials.

PAGE 158 An equal sign is missing from the very end of Solution 4.10.21: $\frac{\pi}{2}\left(\frac{\pi}{2} - \frac{\pi}{8}\right)\frac{3\pi^2}{16}$ should be $\frac{\pi}{2}\left(\frac{\pi}{2} - \frac{\pi}{8}\right) = \frac{3\pi^2}{16}$.

PAGE 165 Solution 4.11, part b: The solution is off by a factor of 4; the integral is

$$4\int_1^2\left(\int_1^2 (x^2+y^2)\,dx\right)dy = 4\int_1^2\left(\int_1^2 (x^2+y^2)dy\right)dx = \frac{56}{3}.$$

PAGE 186 Solution 5.4.1: An integral is missing from the displayed equation, and there are unnecessary absolute values. In the existing integral, $\pi/2$ should be the upper limited, not part of the lower limit. The end of the solution to part b should read:

Thus the image of the Gauss map has area

$$\int_0^{2\pi}\int_{\arcsin\frac{a}{\sqrt{1+a^2}}}^{\frac{\pi}{2}} \cos\varphi\,d\varphi\,d\theta = 2\pi\left[\sin\varphi\right]_{\arcsin\frac{a}{\sqrt{1+a^2}}}^{\frac{\pi}{2}} = 2\pi\left(1-\frac{a}{\sqrt{1+a^2}}\right).$$

PAGE 196 Last line of Solution 6.2.1, part b, minus sign, not plus:

$$\frac{64}{45}-\frac{4}{3}\ln 3, \qquad \text{not} \qquad \frac{64}{45}+\frac{4}{3}\ln 3.$$

PAGE 200 Solution 6.3.5: In the third displayed equation, to respect the order in which the derivative is computed, we should have written $\operatorname{sgn}(-1-1+1)$, not $\operatorname{sgn}(-1+1-1)$.

In the fourth displayed equation, $\operatorname{sgn}(-1+1-1)$ should be $\operatorname{sgn}(-1-1-1)$. Of course $\operatorname{sgn}(-1+1-1) = \operatorname{sgn}(-1-1-1) = -1$, so the mistake does not affect the conclusion.

PAGE 205 Solution 6.4.7, part a: T is neither orientation preserving nor orientation reversing.

PAGE 210 Solution 6.5.17: The last integral is from b to 0, not from a to 0:

$$\int_b^0 \begin{bmatrix}0\\0\end{bmatrix}\cdot\begin{bmatrix}1\\0\end{bmatrix}dt = 0.$$

PAGE 211 In the equation completing Solution 6.5.19, there should be no absolute values around $d\phi$ and $d\theta$.

PAGE 235 Solution 6.11.3: In the first two integrals, the numerator should be ds, not dz. In the first line of the last displayed equation, $pi/2$ should be $\pi/2$

PAGE 239 Solution 6.12.7: In the first line of equation 3, the 2-form on the right should be $dx \wedge dy$, not $dy \wedge dt$.

PAGE 242 Solution 6.7, part b: The first two lines would be better as

The 3-form

$$\Omega_{\mathbf{x}} : (\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2, \vec{\mathbf{v}}_3) \mapsto \det[\mathbf{x}, \vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2, \vec{\mathbf{v}}_3].$$

Two lines later, $\omega_{\mathbf{x}}$ should be $\Omega_{\mathbf{x}}$.

Notes and amplifications

PAGE 114 Here is a second solution to Exercise 3.8.1:

The ellipse of equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \tag{1}$$

is the union of the graphs of

$$f(x) = \frac{b}{a}\sqrt{a^2 - x^2} \quad \text{and} \quad g(x) = -\frac{b}{a}\sqrt{a^2 - x^2}.$$

(It is easier not to think of f explicitly but to think of equation (1) as expressing y implicitly as a function of x.) Clearly the curvatures at $\begin{pmatrix} x \\ f(x) \end{pmatrix}$ and $\begin{pmatrix} x \\ g(x) \end{pmatrix}$ are equal, so we will treat only f. By implicit differentiation, we find

$$y' = -\frac{b^2}{a^2}\frac{x}{y} \quad \text{and} \quad y'' = -\frac{b^4}{a^2y^3},$$

leading to

$$\kappa(x) = \frac{|y''|}{\big(1 + (y')^2\big)^{3/2}} = \frac{b^4}{a^2y^3}\frac{1}{\big(1 + \frac{b^4}{a^4}\frac{x^2}{y^2}\big)^{3/2}} = \frac{a^4b^4}{\big(a^4y^2 + b^4x^2\big)^{3/2}}. \tag{2}$$

If you substitute

$$y^2 = \frac{b^2}{a^2}(a^2 - x^2)$$

in equation (2), you find

$$\kappa = \frac{ba^4}{\big(a^4 - x^2(a^2 - b^2)\big)^{3/2}},$$

as in the first solution.

Minor typos

PAGES 60-61 In the last equation on page 60 and the first equation on page 61, the D should be $\mathbf{D}$.

PAGE 113 Mid-page: There should be a period before the sentence starting "The derivative is".

PAGE 219 Solution 6.7.9: In the first line of the first equation d should be $\mathbf{d}$:

$$\mathbf{d}\omega = \mathbf{d}\Big(p(y,z)\,dx + q(x,z)\,dy\Big)$$

PAGE 243 Solution 6.11: The two $\vec{\gamma}$ (one halfway down the solution, the other in the last line) should be γ, without the arrow.

PAGE 245 Solution 6.15, 2 lines after the first displayed equation, x should be $\mathbf{x}$ in $[\mathbf{Df}(\mathbf{x})]$.

PAGES 247–248 Solution 6.21: All instances of $S_a(0)$ should be $S_a(\mathbf{0})$; all $S_1(0)$ should be $S_1(\mathbf{0})$.

PAGE 250 Solution 6.31, 4 lines from the bottom of the page: $d(\frac{1}{r})$ should be $\mathbf{d}(\frac{1}{r})$.

PAGE 277 Solution A24.1: in the displayed equation, df should be $\mathbf{d}f$.