Student Solution Manual
Vector Calculus, Linear Algebra, and Differential Forms: A Unified Approach, 4th Edition

Notes and Errata

Complete as of May 5, 2012

Many thanks to Alex Huang, Chris Kellner, Jimmy Ojalvo, and Ana Moura Santos for their contributions to this list.

Errors

Page 15 Solution 1.4.19, part b should be " ... which tends to $\pi/2$ as $n \to \infty$."

Page 21 Solution 1.5.21: The solution to part (d) should be

When $0 < x^2 + 2y^2 < 1$,

$$0 > (x^2+y^2)\ln(x^2+2y^2) \geq (x^2+y^2)\ln\big((x^2+y^2)\big),$$

which tends to 0 using the equation in the margin. So if we choose $f\begin{pmatrix}0\\0\end{pmatrix} = 0$, then f is continuous.

Page 29 Solution 1.9.1, last displayed equation:

$$8\frac{(x^2+y^2)^{5/2}}{(x^2+y^2)^2} = 8\sqrt{x^2+y^2}, \quad \text{not } \sqrt{x^2+y^2}.$$

Solution 1.9.3, part a: The equation should be

$$\lim_{\vec{\mathbf{h}}\to\mathbf{0}} \frac{\dfrac{\sin(h_1^2h_2^2)}{h_1^2+h_2^2} - ah_1 - bh_2}{(h_1^2+h_2^2)^{1/2}} = 0.$$

Page 30 Solution 1.9.3, part c: This is wrong, and should be replaced by

Since $D_1f(0) = 0$ and $D_2f(0) = 0$, we see that f is differentiable at the origin if and only if

$$\lim_{|\mathbf{h}|\to 0} \frac{\sin(h_2^2h_2^2)}{(h_1^2+h_2^2)(h_1^2+h_2^2)^{1/2}} = 0,$$

and this is indeed true, since

$$\left|\sin(h_2^2h_2^2)\right| \leq h_1^2h_2^2 \leq \frac{1}{4}\left(h_1^2+h_2^2\right)^2 \qquad (1)$$

and

$$\lim_{|\mathbf{h}|\to 0} \frac{1}{4}\frac{(h_1^2+h_2^2)^2}{(h_1^2+h_2^2)^{3/2}} = \frac{1}{4}\lim_{|\mathbf{h}|\to 0}(h_1^2+h_2^2)^{1/2} = 0.$$

Equation (1), first inequality: For any x, we have $|\sin x| \leq |x|$. The secod inequality follows from

$$0 \leq (x-y)^2 = x^2 - 2xy + y^2,$$

so for any $x, y \in \mathbb{R}$ we have

$$|xy| \leq \frac{1}{2}(x^2+y^2).$$

PAGE 50 The solution to part (b) of exercise 2.5.11 is wrong. Here is a correction.

2.5.11 a. If $ab \neq 2$, then $\dim(\ker\,(A)) = 0$, so in that case the image has dimension 2. If $ab = 2$, the image and the kernel have dimension 1.

b. By row operations, we can bring the matrix B to

$$\begin{bmatrix} 1 & 2 & a \\ 0 & b & ab-a \\ 0 & 2a-b & a^2-a \end{bmatrix}.$$

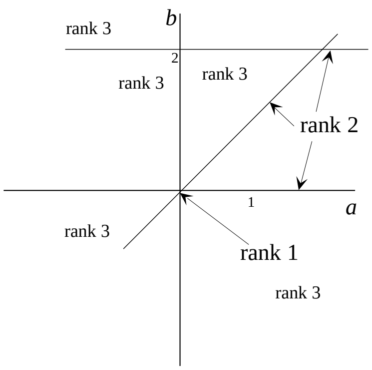
We now separate the case $b \neq 0$ and $b = 0$.

• If $b = 0$, the matrix is $\begin{bmatrix} 1 & 2 & a \\ 0 & 0 & -a \\ 0 & 2a & a^2-a \end{bmatrix}$, which has rank 3 unless $a = 0$, in which case it has rank 1. (Of course, since $n = 3$, rank 3 corresponds to $\dim\ker = 0$, and rank 1 corresponds to $\dim\ker = 2$.)

• If $b \neq 0$, then we can do further row operations to bring the matrix to the form

$$\begin{bmatrix} 1 & 2 & a \\ 0 & b & ab-a \\ 0 & 0 & a^2-a-\frac{2a-b}{b}(ab-a) \end{bmatrix}.$$

The third entry in the third row factors as $\frac{a}{b}(a-b)(2-b)$, so we have rank 2 (and $\dim\ker = 1$) if $a = 0$ or $a = b$ or $b = 2$. Otherwise we have rank 3 (and $\dim\ker = 0$).



CORRECTED FIGURE

Solution 2.5.11, part (b). On the a-axis, on the line $a = b$, and on the line $b = 2$, the image of B has dimension 2, i.e., its kernel has dimension 1. At the origin the rank is 1 and the dimension of the kernel is 2. Elsewhere, the kernel has dimension 0 and the rank is 3.

PAGE 57 Solution 2.7.5. The recursive formula for the b_n is

$$\begin{bmatrix} b_n \\ b_{n+1} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} b_{n-1} \\ b_n \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix}^n \begin{bmatrix} b_0 \\ b_1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix}^n \begin{bmatrix} 1 \\ 1 \end{bmatrix};$$

in the manual, the power n was missing on the matrix $\begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix}$. A basis of eigenvectors is

$$\begin{bmatrix} 1 \\ 1+\sqrt{2} \end{bmatrix}, \quad \begin{bmatrix} 1 \\ 1-\sqrt{2} \end{bmatrix}.$$

PAGE 68 Solution 2.10.9: The derivative given is wrong. It is

$$[\mathbf{D}F(\mathbf{0})] = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}.$$

PAGE 81 Solution 3.1.1: The drivative of $F\begin{pmatrix} s \\ y \\ z \end{pmatrix}$, not the derivative of $F\begin{pmatrix} x \\ y \end{pmatrix}$.

Solution 3.1.7, last displayed equation on the page: $\begin{pmatrix} 1/2 \\ \pm 1/\sqrt{3} \\ z \end{pmatrix}$ (in the second entry, $1/\sqrt{3}$, not $1\sqrt{3}$).

PAGE 82 Solution 3.1.7: "A the points $\begin{pmatrix} 1/2 \\ +1/\sqrt{3} \\ z \end{pmatrix}$, $\begin{pmatrix} 1/2 \\ -1/\sqrt{3} \\ z \end{pmatrix}$".

Solution 3.1.11, part b: $x = \dfrac{y^2}{z}$, not $x = \dfrac{y^2}{x}$

PAGE 89 Solution 3.2.5: In the second and third equations following "which expand to", aA^2 should be a^2A and bB^2 should be b^2B. The correct value for $\mathbf{q}$ is $\mathbf{q} = \begin{pmatrix} a/2 \\ b/2 \\ 0 \end{pmatrix}$, and the volume of the tetrahedron is

$$\frac{1}{6}\left|\det \begin{bmatrix} a & 0 & a/2 \\ 0 & b & b/2 \\ a^2A & b^2B & 0 \end{bmatrix}\right| = \frac{ab}{12}(a^2A + b^2B).$$

PAGE 105 Solution 3.67: In the equation for $D_2^2 f$, $4y^2$ should be $4y^4$:

$$D_2^2 f \begin{pmatrix} x \\ y \end{pmatrix} = (2 - 2x^2 - 10y^2 + 4x^2y^2 + 4y^4)e^{(x^2-y^2)}$$

PAGE 107 Solution 3.7.5, last line: The total volume is $4\sqrt{3}$, not $4\sqrt{2}$.

PAGE 165 Solution 4.11, part b: The solution is off by a factor of 4; the integral is

$$4\int_1^2 \left(\int_1^2 (x^2 + y^2)\,dx\right) dy = 4\int_1^2 \left(\int_1^2 (x^2 + y^2) dy\right) dx = \frac{56}{3}.$$

PAGE 200 Solution 6.3.5: In the third displayed equation, to respect the order in which the derivative is computed, we should have written $\operatorname{sgn}(-1-1+1)$, not $\operatorname{sgn}(-1+1-1)$.

In the fourth displayed equation, $\operatorname{sgn}(-1+1-1)$ should be $\operatorname{sgn}(-1-1-1)$. Of course $\operatorname{sgn}(-1+1-1) = \operatorname{sgn}(-1-1-1) = -1$, so the mistake does not affect the conclusion.

Notes and amplifications

PAGE 114 Here is a second solution to exercise 3.8.1:

The ellipse of equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad (1)$$

is the union of the graphs of

$$f(x) = \frac{b}{a}\sqrt{a^2 - x^2} \quad \text{and} \quad g(x) = -\frac{b}{a}\sqrt{a^2 - x^2}.$$

(It is easier not to think of f explicitly but to think of equation (1) as expressing y implicitly as a function of x.) Clearly the curvatures at $\begin{pmatrix} x \\ f(x) \end{pmatrix}$ and $\begin{pmatrix} x \\ g(x) \end{pmatrix}$ are equal, so we will treat only f. By implicit differentiation, we find

$$y' = -\frac{b^2}{a^2}\frac{x}{y} \quad \text{and} \quad y'' = -\frac{b^4}{a^2 y^3},$$

leading to

$$\kappa(x) = \frac{|y''|}{\big(1 + (y')^2\big)^{3/2}} = \frac{b^4}{a^2 y^3}\frac{1}{\big(1 + \frac{b^4}{a^4}\frac{x^2}{y^2}\big)^{3/2}} = \frac{a^4 b^4}{\big(a^4 y^2 + b^4 x^2\big)^{3/2}}. \qquad (2)$$

If you substitute

$$y^2 = \frac{b^2}{a^2}(a^2 - x^2)$$

in equation (2), you find

$$\kappa = \frac{ba^4}{\big(a^4 - x^2(a^2 - b^2)\big)^{3/2}},$$

as in the first solution.

Typos

PAGES 60-61 In the last equation on page 60 and the first equation on page 61, the D should be $\mathbf{D}$.

PAGE 113 Mid-page: There should be a period before the sentence starting “The derivative is”.

PAGE 219 Solution 6.7.9: In the first line of the first equation d should be $\mathbf{d}$:

$$\mathbf{d}\omega = \mathbf{d}\Big(p(y, z)\, dx + q(x, z)\, dy\Big)$$

PAGE 245 Solution 6.15, 2 lines after the first displayed equation, x should be $\mathbf{x}$ in $[\mathbf{Df}(\mathbf{x})]$.

PAGES 247–248 Solution 6.21: All instances of $S_a(0)$ should be $S_a(\mathbf{0})$; all $S_1(0)$ should be $S_1(\mathbf{0})$.

PAGE 250 Solution 6.31, 4 lines from the bottom of the page: $d(\frac{1}{r})$ should be $\mathbf{d}(\frac{1}{r})$.

PAGE 277 Solution A24.1: in the displayed equation, df should be $\mathbf{d}f$.