Student Solution Manual for the 4th Edition of Vector Calculus, Linear Algebra, and Differential Forms: A Unified Approach

Notes and Errata

Complete as of February 20, 2011

Many thanks to Alex Huang for his contribution to this list.

Errors

PAGE 29 Solution 1.9.3, part a: The equation should be

$$\lim_{\vec{\mathbf{h}} \to \mathbf{0}} \frac{\dfrac{\sin(h_1^2 h_2^2)}{h_1^2 + h_2^2} - ah_1 - bh_2}{(h_1^2 + h_2^2)^{1/2}} = 0.$$

PAGE 30 Solution 1.9.3, part c: This is wrong, and should be replaced by

Since $D_1 f(0) = 0$ and $D_2 f(0) = 0$, we see that f is differentiable at the origin if and only if

$$\lim_{|\mathbf{h}| \to 0} \frac{\sin(h_2^2 h_2^2)}{(h_1^2 + h_2^2)(h_1^2 + h_2^2)^{1/2}} = 0,$$

and this is indeed true, since

$$\left|\sin(h_2^2 h_2^2)\right| \le h_1^2 h_2^2 \le \frac{1}{4}\left(h_1^2 + h_2^2\right)^2 \tag{1}$$

and

$$\lim_{|\mathbf{h}| \to 0} \frac{1}{4} \frac{(h_1^2 + h_2^2)^2}{(h_1^2 + h_2^2)^{3/2}} = \frac{1}{4} \lim_{|\mathbf{h}| \to 0} (h_1^2 + h_2^2)^{1/2} = 0.$$

Equation (1), first inequality: For any x, we have $|\sin x| \le |x|$. The secod inequality follows from

$$0 \le (x-y)^2 = x^2 - 2xy + y^2,$$

so for any $x, y \in \mathbb{R}$ we have

$$|xy| \le \frac{1}{2}(x^2 + y^2).$$

The correction for page 50 is new.

PAGE 50 The solution to part (b) of exercise 2.5.11 is wrong. Here is a correction.

2.5.11 a. If $ab \ne 2$, then dim(ker (A)) $= 0$, so in that case the image has dimension 2. If $ab = 2$, the image and the kernel have dimension 1.

b. By row operations, we can bring the matrix B to

$$\begin{bmatrix} 1 & 2 & a \\ 0 & b & ab - a \\ 0 & 2a - b & a^2 - a \end{bmatrix}.$$

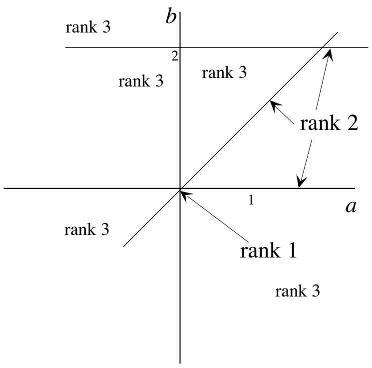
We now separate the case $b \neq 0$ and $b = 0$.

• If $b = 0$, the matrix is $\begin{bmatrix} 1 & 2 & a \\ 0 & 0 & -a \\ 0 & 2a & a^2 - a \end{bmatrix}$, which has rank 3 unless $a = 0$, in which case it has rank 1. (Of course, since $n = 3$, rank 3 corresponds to $\dim \ker = 0$, and rank 1 corresponds to $\dim \ker = 2$.)

• If $b \neq 0$, then we can do further row operations to bring the matrix to the form

$$\begin{bmatrix} 1 & 2 & a \\ 0 & b & ab - a \\ 0 & 0 & a^2 - a - \frac{2a-b}{b}(ab - a) \end{bmatrix}.$$

The third entry in the third row factors as $\frac{a}{b}(a-b)(2-b)$, so we have rank 2 (and $\dim \ker = 1$) if $a = 0$ or $a = b$ or $b = 2$. Otherwise we have rank 3 (and $\dim \ker = 0$).



Corrected figure

Solution 2.5.11, part (b). On the a-axis, on the line $a = b$, and on the line $b = 2$, the image of B has dimension 2, i.e., its kernel has dimension 1. At the origin the rank is 1 and the dimension of the kernel is 2. Elsewhere, the kernel has dimension 0 and the rank is 3.

Page 200   Solution 6.3.5: In the third displayed equation, to respect the order in which the derivative is computed, we should have written $\operatorname{sgn}(-1-1+1)$, not $\operatorname{sgn}(-1+1-1)$.

In the fourth displayed equation, $\operatorname{sgn}(-1+1-1)$ should be $\operatorname{sgn}(-1-1-1)$. Of course $\operatorname{sgn}(-1+1-1) = \operatorname{sgn}(-1-1-1) = -1$, so the mistake does not affect the conclusion.

Notes and amplifications

Page 114   Here is a second solution to exercise 3.8.1:

The ellipse of equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \tag{1}$$

is the union of the graphs of

$$f(x) = \frac{b}{a}\sqrt{a^2 - x^2} \quad \text{and} \quad g(x) = -\frac{b}{a}\sqrt{a^2 - x^2}.$$

(It is easier not to think of f explicitly but to think of equation (1) as expressing y implicitly as a function of x.) Clearly the curvatures at $\begin{pmatrix} x \\ f(x) \end{pmatrix}$ and $\begin{pmatrix} x \\ g(x) \end{pmatrix}$ are equal, so we will treat only f. By implicit differentiation, we find

$$y' = -\frac{b^2}{a^2}\frac{x}{y} \quad \text{and} \quad y'' = -\frac{b^4}{a^2 y^3},$$

leading to

$$\kappa(x) = \frac{|y''|}{\big(1 + (y')^2\big)^{3/2}} = \frac{b^4}{a^2 y^3} \frac{1}{\big(1 + \frac{b^4}{a^4}\frac{x^2}{y^2}\big)^{3/2}} = \frac{a^4 b^4}{\big(a^4 y^2 + b^4 x^2\big)^{3/2}}. \tag{2}$$

If you substitute

$$y^2 = \frac{b^2}{a^2}(a^2 - x^2)$$

in equation (2), you find

$$\kappa = \frac{ba^4}{\left(a^4 - x^2(a^2 - b^2)\right)^{3/2}},$$

as in the first solution.

Typos

PAGES 60-61 In the last equation on page 60 and the first equation on page 61, the D should be $\mathbf{D}$.

PAGE 113 Mid-page: There should be a period before the sentence starting "The derivative is".

PAGE 219 Solution 6.7.9: In the first line of the first equation d should be $\mathbf{d}$:

$$\mathbf{d}\omega = \mathbf{d}\Big(p(y,z)\,dx + q(x,z)\,dy\Big)$$

PAGE 245 Solution 6.15, 2 lines after the first displayed equation, x should be $\mathbf{x}$ in $[\mathbf{Df}(\mathbf{x})]$.

PAGES 247–248 Solution 6.21: All instances of $S_a(0)$ should be $S_a(\mathbf{0})$; all $S_1(0)$ should be $S_1(\mathbf{0})$.

PAGE 250 Solution 6.31, 4 lines from the bottom of the page: $d(\frac{1}{r})$ should be $\mathbf{d}(\frac{1}{r})$.

PAGE 277 Solution A24.1: in the displayed equation, df should be $\mathbf{d}f$.