

STUDENT SOLUTION MANUAL

VECTOR CALCULUS, LINEAR ALGEBRA, AND DIFFERENTIAL FORMS:

A UNIFIED APPROACH, 4TH EDITION

NOTES AND ERRATA

Complete as of Jan. 19, 2013

Errors

PAGE 2 Solution 0.5.1: Three lines from the end of the solution the inequality for $p(-A)$ should be

$$p(-A) \leq (-A)^d + (A-1)A^{d-1} < 0.$$

Many thanks to Alex Huang, Chris Kellner, Jimmy Ojalvo, Ana Moura Santos, and Martin Schmidt for their contributions to this list.

PAGE 15 Solution 1.4.19, part b should be "... which tends to $\pi/2$ as $n \rightarrow \infty$."

PAGE 21 Solution 1.5.21: The solution to part (d) should be

When $0 < x^2 + 2y^2 < 1$,

$$0 > (x^2 + y^2) \ln(x^2 + 2y^2) \geq (x^2 + y^2) \ln((x^2 + y^2)),$$

which tends to 0 using the equation in the margin. So if we choose $f\left(\begin{smallmatrix} 0 \\ 0 \end{smallmatrix}\right) = 0$, then f is continuous.

PAGE 29 Solution 1.9.1, last displayed equation:

$$8 \frac{(x^2 + y^2)^{5/2}}{(x^2 + y^2)^2} = 8\sqrt{x^2 + y^2}, \quad \text{not } = \sqrt{x^2 + y^2}.$$

Solution 1.9.3, part a: The equation should be

$$\lim_{\vec{h} \rightarrow \mathbf{0}} \frac{\frac{\sin(h_1^2 h_2^2)}{h_1^2 + h_2^2} - ah_1 - bh_2}{(h_1^2 + h_2^2)^{1/2}} = 0.$$

PAGE 30 Solution 1.9.3, part c: This is wrong, and should be replaced by

Since $D_1 f(0) = 0$ and $D_2 f(0) = 0$, we see that f is differentiable at the origin if and only if

$$\lim_{|\mathbf{h}| \rightarrow 0} \frac{\sin(h_1^2 h_2^2)}{(h_1^2 + h_2^2)(h_1^2 + h_2^2)^{1/2}} = 0,$$

and this is indeed true, since

Equation (1), first inequality:
 For any x , we have $|\sin x| \leq |x|$.
 The second inequality follows from

$$0 \leq (x - y)^2 = x^2 - 2xy + y^2, \quad \text{and}$$

so for any $x, y \in \mathbb{R}$ we have

$$|xy| \leq \frac{1}{2}(x^2 + y^2).$$

$$\left| \sin(h_1^2 h_2^2) \right| \leq h_1^2 h_2^2 \leq \frac{1}{4} (h_1^2 + h_2^2)^2 \quad (1)$$

$$\lim_{|\mathbf{h}| \rightarrow 0} \frac{1}{4} \frac{(h_1^2 + h_2^2)^2}{(h_1^2 + h_2^2)^{3/2}} = \frac{1}{4} \lim_{|\mathbf{h}| \rightarrow 0} (h_1^2 + h_2^2)^{1/2} = 0.$$

PAGE 50 The solution to part b of exercise 2.5.11 is wrong. Here is a correction.

2.5.11 a. If $ab \neq 2$, then $\dim(\ker(A)) = 0$, so in that case the image has dimension 2. If $ab = 2$, the image and the kernel have dimension 1.

b. By row operations, we can bring the matrix B to

$$\begin{bmatrix} 1 & 2 & a \\ 0 & b & ab - a \\ 0 & 2a - b & a^2 - a \end{bmatrix}.$$

We now separate the case $b \neq 0$ and $b = 0$.

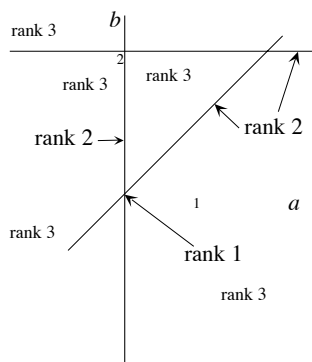
• If $b = 0$, the matrix is $\begin{bmatrix} 1 & 2 & a \\ 0 & 0 & -a \\ 0 & 2a & a^2 - a \end{bmatrix}$, which has rank 3 unless $a = 0$,

in which case it has rank 1. (Of course, since $n = 3$, rank 3 corresponds to $\dim \ker = 0$, and rank 1 corresponds to $\dim \ker = 2$.)

• If $b \neq 0$, then we can do further row operations to bring the matrix to the form

$$\begin{bmatrix} 1 & 2 & a \\ 0 & b & ab - a \\ 0 & 0 & a^2 - a - \frac{2a-b}{b}(ab - a) \end{bmatrix}.$$

The third entry in the third row factors as $\frac{a}{b}(a-b)(2-b)$, so we have rank 2 (and $\dim \ker = 1$) if $a = 0$ or $a = b$ or $b = 2$. Otherwise we have rank 3 (and $\dim \ker = 0$).



CORRECTED FIGURE

Solution 2.5.11, part (b). On the a -axis, on the line $a = b$, and on the line $b = 2$, the image of B has dimension 2, i.e., its kernel has dimension 1. At the origin the rank is 1 and the dimension of the kernel is 2. Elsewhere, the kernel has dimension 0 and the rank is 3.

PAGE 57 Solution 2.7.5. The recursive formula for the b_n is

$$\begin{bmatrix} b_n \\ b_{n+1} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} b_{n-1} \\ b_n \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix}^n \begin{bmatrix} b_0 \\ b_1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix}^n \begin{bmatrix} 1 \\ 1 \end{bmatrix};$$

in the manual, the power n was missing on the matrix $\begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix}$. A basis of eigenvectors is

$$\begin{bmatrix} 1 \\ 1 + \sqrt{2} \end{bmatrix}, \quad \begin{bmatrix} 1 \\ 1 - \sqrt{2} \end{bmatrix}.$$

PAGE 68 Solution 2.10.9: The derivative given is wrong. It is

$$[DF(\mathbf{0})] = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}.$$

PAGE 81 Solution 3.1.1: The derivative of $F \begin{pmatrix} x \\ y \\ z \end{pmatrix}$, not the derivative of $F \begin{pmatrix} x \\ y \end{pmatrix}$.

Solution 3.1.5, 3 lines before the end of part a: $y = -x^{2/3}$, not $y = x^{2/3}$

Solution 3.1.7, last displayed equation on the page: $\begin{pmatrix} 1/2 \\ \pm 1/\sqrt{3} \\ z \end{pmatrix}$ (in the second entry, $1/\sqrt{3}$, not $1\sqrt{3}$).

PAGE 82 Solution 3.1.7: “A the points $\begin{pmatrix} 1/2 \\ +1/\sqrt{3} \\ z \end{pmatrix}$, $\begin{pmatrix} 1/2 \\ -1/\sqrt{3} \\ z \end{pmatrix}$ ”.

Solution 3.1.11, part b: $x = \frac{y^2}{z}$, not $x = \frac{y^2}{x}$

PAGE 89 Solution 3.2.5: In the second and third equations following “which expand to”, aA^2 should be a^2A and bB^2 should be b^2B . The correct value for \mathbf{q} is $\mathbf{q} = \begin{pmatrix} a/2 \\ b/2 \\ 0 \end{pmatrix}$, and the volume of the tetrahedron is

$$\frac{1}{6} \left| \det \begin{bmatrix} a & 0 & a/2 \\ 0 & b & b/2 \\ a^2A & b^2B & 0 \end{bmatrix} \right| = \frac{ab}{12}(a^2A + b^2B).$$

PAGE 105 Solution 3.6.7: In the equation for D_2^2f , $4y^2$ should be $4y^4$:

$$D_2^2f \begin{pmatrix} x \\ y \end{pmatrix} = (2 - 2x^2 - 10y^2 + 4x^2y^2 + 4y^4)e^{(x^2-y^2)}$$

PAGE 107 Solution 3.7.5, last line: The total volume is $4\sqrt{3}$, not $4\sqrt{2}$.

PAGE 165 Solution 4.11, part b: The solution is off by a factor of 4; the integral is

$$4 \int_1^2 \left(\int_1^2 (x^2 + y^2) dx \right) dy = 4 \int_1^2 \left(\int_1^2 (x^2 + y^2) dy \right) dx = \frac{56}{3}.$$

PAGE 196 Last line of solution 6.2.1, part b, minus sign, not plus:

$$\frac{64}{45} - \frac{4}{3} \ln 3, \quad \text{not} \quad \frac{64}{45} + \frac{4}{3} \ln 3.$$

PAGE 200 Solution 6.3.5: In the third displayed equation, to respect the order in which the derivative is computed, we should have written $\text{sgn}(-1 - 1 + 1)$, not $\text{sgn}(-1 + 1 - 1)$.

In the fourth displayed equation, $\text{sgn}(-1+1-1)$ should be $\text{sgn}(-1-1-1)$. Of course $\text{sgn}(-1+1-1) = \text{sgn}(-1-1-1) = -1$, so the mistake does not affect the conclusion.

PAGE 210 Solution 6.5.17: The last integral is from b to 0, not from a to 0:

$$\int_b^0 \begin{bmatrix} 0 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} dt = 0.$$

PAGE 211 In the equation completing solution 6.5.19, there should be no absolute values around $d\phi$ and $d\theta$.

Notes and amplifications

PAGE 114 Here is a second solution to exercise 3.8.1:

The ellipse of equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \tag{1}$$

is the union of the graphs of

$$f(x) = \frac{b}{a}\sqrt{a^2 - x^2} \quad \text{and} \quad g(x) = -\frac{b}{a}\sqrt{a^2 - x^2}.$$

(It is easier not to think of f explicitly but to think of equation (1) as expressing y implicitly as a function of x .) Clearly the curvatures at $\begin{pmatrix} x \\ f(x) \end{pmatrix}$ and $\begin{pmatrix} x \\ g(x) \end{pmatrix}$ are equal, so we will treat only f . By implicit differentiation, we find

$$y' = -\frac{b^2 x}{a^2 y} \quad \text{and} \quad y'' = -\frac{b^4}{a^2 y^3},$$

leading to

$$\kappa(x) = \frac{|y''|}{(1 + (y')^2)^{3/2}} = \frac{b^4}{a^2 y^3} \frac{1}{(1 + \frac{b^4 x^2}{a^4 y^2})^{3/2}} = \frac{a^4 b^4}{(a^4 y^2 + b^4 x^2)^{3/2}}. \tag{2}$$

If you substitute

$$y^2 = \frac{b^2}{a^2}(a^2 - x^2)$$

in equation (2), you find

$$\kappa = \frac{ba^4}{(a^4 - x^2(a^2 - b^2))^{3/2}},$$

as in the first solution.

Typos

PAGES 60-61 In the last equation on page 60 and the first equation on page 61, the D should be \mathbf{D} .

PAGE 113 Mid-page: There should be a period before the sentence starting “The derivative is”.

PAGE 219 Solution 6.7.9: In the first line of the first equation d should be \mathbf{d} :

$$\mathbf{d}\omega = \mathbf{d}\left(p(y, z) dx + q(x, z) dy\right)$$

PAGE 245 Solution 6.15, 2 lines after the first displayed equation, x should be \mathbf{x} in $[\mathbf{Df}(\mathbf{x})]$.

PAGES 247–248 Solution 6.21: All instances of $S_a(0)$ should be $S_a(\mathbf{0})$; all $S_1(0)$ should be $S_1(\mathbf{0})$.

PAGE 250 Solution 6.31, 4 lines from the bottom of the page: $d(\frac{1}{r})$ should be $\mathbf{d}(\frac{1}{r})$.

PAGE 277 Solution A24.1: in the displayed equation, df should be $\mathbf{d}f$.